Low-Cost Distributed Massive MIMO System: Achievable Rate and Energy Efficiency

Jide Yuan, Qi Zhang, Tony Q. S. Quek, Chao-Kai Wen, and Shi Jin

Abstract—This paper proposes a low-cost distributed massive multiple-input multiple-output (MIMO) system, which employs the mixed analog-to-digital converter (ADC) remote radio heads (RRHs). In particular, the RRHs with low-resolution ADCs connect with the baseband unit through wireless fronthaul while the RRHs with full-resolution ADCs connect with the baseband unit through fiber. After estimating the channel state information between RRHs and users, we derive the closed-form expressions for achievable downlink rate and energy efficiency. Based on these analytical results, we find that our distributed architecture can achieve the rate requirement that can maximize the energy efficiency under a fixed total number of RRHs, which can be used as guidelines for practical network configuration.

Index Terms—Achievable rate, distributed massive MIMO, energy efficiency, hybrid fronthaul, mixed-ADC receiver.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is an anticipated technique in fifth generation (5G) communication which can provide significant improvement in spectral efficiency and energy efficiency (EE) with simple signal processing [1,2]. The distributed layout, in which the antennas are spread out over a large area while connecting with a baseband unit via a fronthaul network, is proposed for its larger coverage and better overall performance [3]. Therefore, a new architecture that massive remote radio heads (RRHs) simultaneously serve dozens of users is proposed in [4], which is called cell-free massive MIMO. The cell-free massive MIMO shows remarkably superior throughputs compared with the centralized massive MIMO and conventional small cell systems, especially when appropriate power control strategy is applied. However, most previous works on distributed layout equip all nodes with high resolution antennas as well as fiber-based fronthaul, which will bring remarkable cost for distributed massive MIMO systems, since the hardware cost grows linearly as the increment of antenna number [4,5].

To handle with the severe hardware cost, the low-resolution analog-to-digital converters (ADCs) are invited to save both the expenditure and energy at antennas [6,7]. Based on it, the mixed-ADC receiver architecture is proposed [8,9], in which a fraction of the antennas equips with the full-resolution ADCs, whereas the rest have low-resolution. In [8], the mixed-ADC receiver is achieved through probabilistic Bayesian inference. On the other hand, the high cost of fiber fronthaul can be addressed with wireless fronthaul. The results in [10] has showed that the wireless fronthaul network can perfectly achieve the system requirement with enough transmit power.

Fig. 1. The system model of low-cost distributed massive MIMO, where F-RRHs and L-RRHs connect with BBU through fiber-based network and wireless network, respectively.

In this paper, we aim to find an architecture for distributed massive MIMO systems that can balance the performance and price. We propose a distributed massive MIMO system with mixed-ADC RRHs and hybrid fronthauls. In particular, the RRHs equipped with full resolution ADCs, named as F-RRHs, connect with the baseband unit (BBU) via fibers, while the RRHs equipped with low resolution, names as L-RRHs, connect with the BBU through wireless path. This is a reasonable framework since the L-RRH is given for cost reduction, it’s natural to equip L-RRH with wireless fronthaul instead of expensive fiber fronthaul. Our aim is to analyze the downlink achievable performance under the assumption that the perfect knowledge of channel statement information of wireless fronthaul (CSIF) and the estimated channel information from user to RRH (CSIU) are known at BBU. After estimating channel state information between RRHs and users, we derive closed-form expressions for

J. Yuan and S. Jin are with the National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, P. R. China (e-mail: {yuanjide, jinshi}@seu.edu.cn).
C. Wen is with the Institute of Communications Engineering, National Sun Yat-sen University, Kaohsiung 804, Taiwan (e-mail: ckwen@ieee.org).
Qi Zhang and Tony Q. S. Quek is with the Singapore University of Technology and Design, Singapore 487382, Singapore (e-mail: qizhangqi@12129126.com; Tonyquek@sutd.edu.sg).
This work was supported in part by the National Science Foundation (NSFC) for Distinguished Young Scholars of China with Grant 61625106 and the NSFC with Grant 61531011.
achievable downlink rate and energy efficiency, where both the power consumption of RRHs and fronthaul network are considered. Based on these analytical results, we find that our proposed distributed massive MIMO system can obtain significant rate gains compared with the centralized layout. Moreover, compared with the conventional distributed MIMO network with pure F-RRHs, the proposed system can achieve the rate requirement in a more energy-efficient and low-cost way, especially for low rate demands. For practical reference, we also present the optimal RRH assignment proportion that can maximize the energy efficiency.

II. SYSTEM MODEL

We consider the downlink of a multi-user low-cost distributed massive MIMO system where $M$ single-antenna RRHs connect to BBU via a hybrid fronthaul network, as shown in Fig. 1. The BBU is established at the cloud side, which equips with $N$ antennas for wireless fronthaul transmission. Among $M$ RRHs, only a proportion of RRHs, say $M_f$, are F-RRHs, while the rest ($M_l = M - M_f$) are L-RRHs. Define $\kappa \triangleq M_l / M$ as the proportion of F-RRHs in all RRHs. For the sake of saving cost, only F-RRHs connect with BBU through fiber fronthaul which can provide unlimited and error-free fronthaul capacity, while the L-RRHs use the wireless path for fronthaul due to the low cost which accords with the purpose of introducing L-RRHs.

In the downlink, $K$ single-antenna users simultaneously served by $M$ RRHs in the same time-frequency resource, where the locations of RRHs and users are randomly generated. Since the introduction of wireless fronthaul, the BBU need to perform beamforming for both the transmission of wireless fronthaul and the access links, which can be realized by collecting channel state information from all RRHs. These two stages of transmissions are assigned with different frequency band to avoid interference. The detailed procedure of beamforming and channel estimation will be described in the following subsections.

A. Channel Estimation

We model the complex propagation coefficient between the antenna $i$ and $j$ as

$$ g_{ij} = h_{ij} \sqrt{\beta_{ij}}, $$

(1)

where $h_{ij}$ is the fast fading coefficient which is assumed to have zero mean and unit variance, and $\beta_{ij}$ is the large scale fading. We denote the channel coefficient from $m$th F-RRHs to $k$th user, from $m'$th L-RRHs to $k$th user and from $m$th antenna at BBU to $m'$th L-RRHs as $g_{l,m,k}$, $g_{l,m',k}$ and $g_{b,m,n,m'}$, respectively. Due to the low mobility of RRHs, we assume the state information of channels between L-RRHs and BBU can be perfectly known at BBU. Therefore, we only need to estimate the channels between RRHs and users.

In uplink training, all $K$ users simultaneously send their pilot sequences to RRHs. Then, the received pilot vector at $m$th F-RRH is

$$ \mathbf{x}_{p,m} = \sqrt{\tau \rho_p} \sum_{k=1}^{K} g_{l,m,k} \mathbf{\varphi}_k + \omega_m, $$

(2)

where $\mathbf{\varphi}_k \in \mathbb{C}^{r \times 1}$ is the pilot sequences from the $k$th users to the $m$th F-RRH with $||\mathbf{\varphi}_k||^2 = 1$. $\omega_m \in \mathbb{C}^{r \times 1}$ is the vector which denotes an additive white Gaussian noise, whose entries are i.i.d. circular complex Gaussian random variables with zero mean, and $\rho_p$ is the transmit signal-to-noise ratio (SNR) of each pilot symbol. From (2), the detected channel information of $k$th user at the $m$th F-RRH can be derived as

$$ \hat{x}_{p,m,k} = \sqrt{\tau \rho_p} g_{l,m,k} + \sqrt{\tau \rho_p} \sum_{k' \neq k} g_{l,m,k'} \mathbf{\varphi}_k^H \mathbf{\varphi}_{k'} + \mathbf{\varphi}_k^H \omega_m. $$

(3)

Based on which, the MMSE estimate of $g_{l,m,k}$ can be further given as

$$ \hat{g}_{l,m,k} = \frac{E[\hat{x}_{p,m,k} \mathbf{\varphi}_k]}{E[||\hat{x}_{p,m,k}||^2]} \hat{x}_{p,m,k} = \hat{c}_{l,m,k} x_{p,m,k}, $$

(4)

where

$$ \hat{c}_{l,m,k} = \frac{\sqrt{\tau \rho_p} \beta_{l,m,k}}{\tau \rho_p \sum_{k'=1}^{K} \beta_{l,m,k'} |\mathbf{\varphi}_{k'}^H \mathbf{\varphi}_{k}|^2 + 1}. $$

(5)

Let $\varepsilon_{l,m,k} = \hat{g}_{l,m,k} - g_{l,m,k}$ be the channel estimation error. Then, the variance of $\varepsilon_{l,m,k}$ can be got as

$$ \sigma_{\varepsilon_{l,m,k}}^2 = E \left( |\varepsilon_{l,m,k} - E[\varepsilon_{l,m,k}]|^2 \right) = \beta_{l,m,k} - \lambda_{l,m,k}, $$

(6)

with $\lambda_{l,m,k} = \sqrt{\tau \rho_p} \hat{c}_{l,m,k} / \beta_{l,m,k}$. For the L-RRHs, each of which is equipped with $b$-bit quantizer, we adopt the additive quantization noise model in [11] to formulate the quantization signal. Hence, the quantization pilot vector at $m$th L-RRH can be depicted as

$$ \hat{x}_{p,m'} = \alpha \mathbf{x}_{p,m'} + \omega_{p,q}, $$

(7)

where $\omega_{p,q}$ is the quantization noise involved in received pilot, $\alpha \in [0,1]$ is a coefficient listed in Table I [11], which varies according to the number of the quantization bits $b$. After similar manipulations, the MMSE estimate of $g_{l,m,k}$ can be derived as

$$ \hat{g}_{l,m,k} = \hat{c}_{l,m,k} x_{p,m',k}, $$

(8)

with

$$ \hat{c}_{l,m,k} = \frac{\alpha \sqrt{\tau \rho_p} \beta_{m,k}}{\alpha^2 \left( \tau \rho_p \sum_{k'=1}^{K} \beta_{m,k'} |\mathbf{\varphi}_{k'}^H \mathbf{\varphi}_{k}|^2 + 1 \right) + C_{\omega_{p,q}}} + C_{\omega_{p,q}}, $$

(9)

where $C_{\omega_{p,q}}$ denotes the variance of quantization noise that is uncorrelated with $x_{p,m'}$, and is given as [7]

$$ C_{\omega_{p,q}} = \alpha (1 - \alpha) \left( 1 + \rho_p \sum_{k'=1}^{K} \beta_{m,k'} \right). $$

(10)
Combing (9) and (8), we can get that
\[ \hat{c}_{l,m,k'} = \frac{1}{\rho_p} \sqrt{\rho_p \gamma_k \beta_{m,k'}}. \]  
(11)

Let \( \varepsilon_{l,m,k'} = \hat{g}_{l,m,k'} - g_{l,m,k} \) be the channel estimation error of \( m' \) th L-RRH. Similarly as (6), we can have that
\[ \sigma^2_{l,m,k'} = \beta_{l,m,k'} - \alpha \lambda_{l,m,k'}. \]  
(12)

with \( \lambda_{l,m,k'} = \sqrt{\rho_p} \gamma_k \beta_{m,k'}. \) Therefore, the overall estimated channel from \( k \) users to all \( M \) RRHs can be packed into a \( K \times M \) matrix as
\[ \hat{G} = [\hat{g}_{l,1,k}, \ldots, \hat{g}_{l,K,k}]^T, \]  
(13)

where \( \hat{g}_{l,k} = [\hat{g}_{l,k,1}, \hat{g}_{l,k,2}, \ldots, \hat{g}_{l,k,L}]. \) \( \hat{g}_{l,k} \in \mathbb{C}^M, \) and \( \hat{g}_{l,k} \in \mathbb{C}^M \) represents the channel propagation vector from \( k \) th user to F-RRHs and L-RRHs, i.e., \( \hat{g}_{l,k} \) is the channel propagation vector from \( k \) th user to L-RRHs.

Then, the transmit vector at F-RRHs can be given as
\[ \mathbf{x}_f = \sum_{k=1}^K [\hat{g}_{l,1,k}^H, \ldots, \hat{g}_{l,K,k}^H] s_k, \]  
(14)

where \( s_k \in \mathbb{C} \) is the data symbol destined for \( k \) th user with unit variance.

For the links through L-RRHs, BBU performs beamforming for both the downlink data transmission from RRHs to users and the transmission from BBU to L-RRHs.

The results reveal the impact of various parameters on the downlink rate. It is obvious that the achievable rate increases with growing \( \rho \) and converges as \( \rho \to \infty \) due to the interference. Moreover, since the L-RRHs amplify the received signals before forwarding them as shown in (18), an appropriate increase on transmit SNR at the wireless fronthaul can effectively improve the signal quality due to the suppression on Gaussian noise. However, this improvement will be limited when \( \rho_B \) grows quite large, since the quantization noise will be proportional to the received signal strength if \( \rho_B \to \infty \) according to (20).

### III. ACHIEVABLE RATE AND ENERGY EFFICIENCY

In this section, we derive the closed-form expression of downlink achievable rate and EE of the proposed distributed massive MIMO system.

### A. Achievable Rate

We consider that user has the statistics knowledge of the channel estimates, according to the classical assumption of worst-case uncorrelated Gaussian noise [4], we derive the downlink achievable rate of \( k \) th user.

**Theorem 1:** In the proposed distributed massive MIMO system, the downlink achievable rate of \( k \) th user is
\[ R_k = \log_2 (1 + \rho_k \text{SINR}_k), \]  
where \( \rho_k \) is given in (19) at the top of the next page, where \( P_{N_k} \) is the normalized noise power given by
\[ P_{N_k} = \alpha^2 \sum_{m'=1}^M \mu_{l,m'}^2 \beta_{m',k'} + \frac{M}{\rho_d}, \]  
and \( P_{QN_k} \) denotes the quantization noise power which is given as
\[ P_{QN_k} = \alpha (1 - \alpha) \sum_{m'=1}^M \lambda_{l,m'} (\mu_{l,m'}^2 \chi_{m'} + 1), \]  
(20)

with \( \mu_{l,i} = \left( \frac{\sum_{k=1}^K \lambda_{l,ik}}{M} \right)^{-\frac{1}{2}}, \gamma_i = \left( \frac{\sum_{k=1}^K \lambda_{l,ik}}{M} \right)^{-\frac{1}{2}}, \mu_{l,i} = \alpha^2 \gamma_i + \alpha (1 - \alpha) \chi_i + \alpha, \chi_i = \rho_B \| v_{B,i} \|, \) and \( v = \rho_B/N_{\text{PHY}}^0. \)

**Proof:** See Appendix.

The results reveal the impact of various parameters on the downlink rate. It is obvious that the achievable rate increases with growing \( \rho \), and converges as \( \rho \to \infty \) due to interference. Moreover, since the L-RRHs amplify the received signals before forwarding them as shown in (18), an appropriate increase on transmit SNR at the wireless fronthaul can effectively improve the signal quality due to the suppression on Gaussian noise. However, this improvement will be limited when \( \rho_B \) grows quite large, since the quantization noise will be proportional to the received signal strength if \( \rho_B \to \infty \) according to (20).
\[
\text{SINR}_k = \frac{\left( \sum_{m=1}^{M} \mu_{m} \lambda_{m} + \alpha^2 \frac{1}{2} \left( \sum_{m'=1}^{M} \gamma_{m'} \mu_{m'} \lambda_{m'} \right) \right)^2}{\sum_{i=k}^{K} \left( \sum_{m=1}^{M} \mu_{i,m} \beta_{m,i} \lambda_{m,i} + \alpha^3 \frac{1}{2} \sum_{m'=1}^{M} \mu_{i,m'} \gamma_{m'} \beta_{m',i} \lambda_{i,m'} \right) + P_{Q_k} + P_{N_k}}
\]

(19)

**B. Energy Efficiency**

EE is an important metric to balance the rate performance and power consumption. Since wireless fronthaul cost much less power than fiber, EE can also be used to balance the rate performance and price in our model. Here, we consider the power consumption of RRHs and hybrid fronthaul networks.

To evaluate the power dissipation of each RRH, we adopt the model proposed in [12], which approximates the power consumption of RRHs and hybrid fronthaul networks. By the total power consumption, which is given as

\[ P_{\text{total}} = \varepsilon \rho_d \sigma_N^2 + P_c, \quad 0 \leq \rho_d \alpha N \leq P_{\text{max}}, \]

(21)

where \( \varepsilon \) is a constant represented for amplifier efficiency, \( \sigma_N^2 \) accounts for the variance of Gaussian noise, \( P_c \) is the wireless fronthaul bandwidth and \( \rho_d \) is a constant represented for amplifier efficiency.

For the hybrid fronthaul network, we adopt different power consumption models for two kinds of fronthaul connections. For the wireless fronthaul, we consider the power consumption of the ADC at L-RRHs and the transmit power at BBU, which is given as

\[ P_{\text{WF}} = P_B + M_i P_{\text{ADC}}, \]

(22)

where

\[ P_B = \varepsilon \rho_B \sigma_N^2, \quad 0 \leq \rho_B \sigma_N^2 \leq P'_{\text{max}}, \]

(23)

is the transmit power from BBU, and

\[ P_{\text{ADC}} = c_0 W_F 2^{2b}, \]

(24)

is the power dissipation of \( b \)-bits ADC associated with antenna at each L-RRH [13], while \( W_F \) is the wireless fronthaul bandwidth and \( c_0 \) is a constant depending on the ADC circuit.

On the other hand, the power consumption of fiber-based fronthaul, which connects the F-RRHs to BBU, can be got from the cable fronthaul model in [5] as follows,

\[ P_{\text{FF}} = \zeta R_{\text{FF}} = \zeta W_d M_f \sum_{k=1}^{K} R_k, \]

(25)

where \( \zeta \) is a constant scaling factor, \( R_{\text{FF}} \) is the fronthaul traffic which equals to the total downlink throughput, and \( W_d \) denoting the downlink bandwidth.

Therefore, the total power consumption in our system can then be calculated as

\[ P_{\text{total}} = \varepsilon \rho_d \sigma_N^2 (M \rho_d + \rho_B) + M P_c + M_i c_0 W_F 2^{2b} + \zeta M_f W_d \sum_{k=1}^{K} R_k, \]

(26)

and the EE is defined as the total throughput of users divided by the total power consumption, which is given as

\[ \psi = \frac{W_d \sum_{k=1}^{K} R_k}{P_{\text{total}}} \text{ (bits/Joule)}. \]

(27)

Next, we evaluate the impact of transmit power and F-RRH proportion on the achievable rate and energy efficiency via numerical results. In particular, for practical configuration reference, we also present the optimal F-RRH proportion that can maximize the energy efficiency while achieving the throughput requirement.

**IV. Numerical Results**

In our simulations, \( M \) RRHs and \( K \) users were randomly distributed in a 1 km² area. The large-scale fading coefficient consists of path loss and shadowing, and is modeled as follows (in \( \text{dB} \)) [14]

\[ \beta(d) = \begin{cases} 30.2 + 23.5 \log_{10}(d), & \text{BBU - RRH,} \\ 32.9 + 36.3 \log_{10}(d), & \text{RRH - user,} \end{cases} \]

where \( d \) is the distance between two particles. The background noise power, wireless fronthaul bandwidth, and downlink bandwidth were assumed to be -174 dbm/Hz, \( W_F = 10 \text{MHz} \), and \( W_d = 10 \text{MHz} \). As for the power consumption parameters, we set \( \varepsilon = 2.8 \), \( P_c = 0.2 \text{W} \), \( c_0 = 2 \times 10^{-4} \), \( \zeta = 0.25 \text{W/(Mbits/s)} \), and \( P'_{\text{max}} = P_{\text{max}} = 43 \text{dBm} \). Furthermore, we assume that the pilot sequences assigned to each user are mutual orthogonal.

We first compare the performance between distributed and centralized massive MIMO systems. From Fig. 2, we can find that the 95%-likely per-user downlink rate in distributed massive MIMO for 10 dB and 20 dB are about 4 Mbits/s and 12 Mbits/s, whereas the 95% users in the centralized layout can hardly get services. The result indicates that the low-resolution ADC receiver may not be suitable for centralized massive MIMO due to its poor-quality channel estimation for the faraway users. However, because of the reduced
transmission distance between RRHs and users, the distributed massive MIMO system can significantly benefit from the channel diversity. Moreover, we see that only 3 bits/s/Hz loss of the proposed architecture compared with the full F-RRHs configuration, which means the low-cost distributed massive MIMO can provide robust services but with a much lower hardware cost.

Fig. 3 evaluates the behavior of downlink rate as respect to the fronthaul transmit power, \( \rho_p \), which determines the received signal strength at L-RRHs. We also illustrate the pure F-RRHs configurations as counterparts, shown in blue lines. As we can see, when \( \rho_p \) is small, the downlink rate increases remarkably as \( \rho_p \) grows, whereas this improvement becomes limited as \( \rho_p \) grows large. Another important observation is that for low \( \rho_p \), the performance for the distributed massive MIMO system with 20-F-RRHs and 80-L-RRHs is similar with that with pure 20-F-RRHs. This is because for low \( \rho_p \), the received fronthaul signal at L-RRH is very weak and dominated by the quantization and Gaussian noise, while these impairments become even severer after the amplifier thus result in significant decline on the rate performance. However, if \( \rho_p \) is larger than 20 dB, the performance of the distributed architecture with 20-L-RRHs with 80-F-RRHs outperforms that with the pure 60F-RRHs configurations, which indicates that the wireless fronthaul transmit power is one of the most decisive factor in mixed-ADC distributed massive MIMO. Under enough wireless fronthaul transmit power, the proposed architecture can achieve the same or even better performance as the network with pure full-resolution RRHs, but the hardware cost is much lower.

Fig. 4 reveals the behavior of EE with respect to the F-RRH proportion \( \kappa \) for fixed transmit power. In this simulation, we set a downlink rate threshold \( T \) (in Mbits/s) to guarantee the requirement of users. It can be found that when \( \kappa \) is very small, EE becomes zero since the configuration with few F-RRHs cannot achieve the rate requirement. As \( \kappa \) grows large, the EE first increases and then reduces, where an optimal \( \kappa \) that maximize the EE can be observed. Compared with the configuration with pure F-RRHs, i.e., \( \kappa = 1 \), the EE with optimal \( \kappa \) can obtain 125\%, 50\%, and 30\% gains for \( T = 30, 60 \) and 90, respectively. This observation indicates that our mixed-ADC distributed massive MIMO system can provide sufficient service with less power consumption as well as less hardware cost.

To provide guidelines for practical network configurations, we present the optimal \( \kappa \) that can maximize the EE in Fig. 5. We can find that the optimal \( \kappa \) increase with \( T \) grows, because a higher rate threshold needs more F-RRHs and the optimal \( \kappa \) equals to 1 at some \( T \) which means only the configuration with pure F-RRHs can reach this rate requirement. Moreover, similar trend can be found among the lines with same \( \rho_d \) and different \( \rho_p \) indicating the low power dissipation of the wireless fronthaul network. Above all, we can conclude that compared with the network with pure F-RRHs, our mixed-ADC distributed massive MIMO system can achieve the rate requirement in a more energy-efficient and low-cost way, especially for low rate demands.
V. Conclusion

A low-cost distributed massive MIMO system employing the mixed ADC antennas and hybrid fronthaul network has been proposed for its high energy efficiency and low-cost implementation. After estimating channel state information between RRHs and users, we have derived closed-form expressions for achievable downlink rate and energy efficiency. Based on these analytical results, we have found that our distributed architecture can achieve significant rate gains compared with the centralized layout, especially with our distributed architecture can achieve significant rate gains compared with the centralized layout, especially with sufficient wireless fronthaul transmit power. More importantly, a remarkable gain on the energy efficiency can be obtained for the proposed architecture, and we also present the optimal RRH assignment proportion that maximizes the energy efficiency for practical configuration reference.

Appendix

Recall that ZF precoding is adopted for wireless fronthaul along with (16), we have

$$g_{B_i}^H v_{B,m'} = \begin{cases} \eta_{B_i}, & i = m', \\ 0, & i \neq m'. \end{cases}$$  (29)

Substituting (14) and (17) into (18), we rewrite the received signal as

$$r_k = \sqrt{\frac{P_0}{M}} \left( \sum_{m=1}^{M_t} \mu_{k,m} \sum_{i=1}^{K} g_{k,m}^H s_i \\
+ \alpha \frac{\sigma^2}{2} \sum_{m=1}^{M_t} \mu_{k,m} \gamma_{m} \sum_{i=1}^{K} g_{k,m}^H s_i \\
+ g_{k,k} (\mu_{k} \cdot \omega_{k}) + \alpha g_{k,k} (\mu_{k} \cdot \omega_{k}) \right) + \omega_k$$

$$= \sqrt{\frac{P_0}{M}} \left( (D_{S,k} + EU_k) s_k + \sum_{i \neq k}^{K} IF_{k,i} s_i + QN_k \right) + N_k,$$  (30)

where $\alpha$ is obtained by adopting (29), and each term represents the interference ($IF_{k,i}$) caused by other users, the quantization noise ($QN_k$) and the AWGN ($N_k$), respectively. Since user $k$ has the knowledge of estimated channel statistics, the first two terms on the right of (30) is separated into desired signals ($D_{S,k}$) of $k$th user, the interference due to the channel estimation uncertainty ($E_{U,k}$), given as

$$DS_k = \sum_{m=1}^{M_t} \mu_{k,m} m E \left( |g_{k,m,k}|^2 \right)$$

$$+ \alpha \frac{\sigma^2}{2} \sum_{m=1}^{M_t} \mu_{k,m,k} \gamma_{m} \sum_{i=1}^{K} g_{k,m,k}^H s_i$$

$$EU_k = \sum_{m=1}^{M_t} \mu_{k,m,k} \gamma_{m} \sum_{i=1}^{K} g_{k,m,k}^H s_i$$

$$+ \alpha \frac{\sigma^2}{2} \sum_{m=1}^{M_t} \mu_{k,m,k} \gamma_{m} \sum_{i=1}^{K} g_{k,m,k}^H s_i$$

It is easy to conclude that the terms $DS_k$, $EU_k$, $IF_{k,i}$, $QN_k$ and $N_k$ are mutually uncorrelated. The SINR of the worst case achievable rate of $k$th user is given as

$$\text{SINR}_k = \frac{|DS_k|^2}{E \left[ |EU_k|^2 \right] + \sum_{i \neq k}^{K} E \left[ |IF_{k,i}|^2 \right] + E \left[ |QN_k|^2 \right] + E \left[ |N_k|^2 \right]}$$

The variances of $DS_k$, $EU_k$, $IF_{k,i}$ and $N_k$ can be computed simply by applying (6), (12), $E \left[ |g_{k,m,k}|^2 \right] = 2 \lambda_{m,k}^2$ and $E \left[ |Q|_{mn,k}^2 \right] = 2 \lambda_{m,k}^2$, along with the fact that the channel estimates error has zero mean and is independent of channel statement.

For the quantization noise, following the results in [11], we have $E \left[ |Q|_{mn,k}^2 \right] = E \left[ |g_{k,m}^H R_{\mu_{k},\omega_{k}} g_{k,m}^H | \right]$, where $R_{\mu_{k},\omega_{k}} = \alpha (1 - \alpha) \sigma^2.$ Note the fact of (29) and (15), we obtain the $m$th diagonal element as $\alpha (1 - \alpha) \frac{\mu_{k,m,k}^2 \gamma_m}{N} \sum_{n=1}^{H} g_{B,n,m}^H g_{B,n,m'} + 1$, and the power of quantization noise can be simply computed using above conclusions. We now calculate the power amplification vectors $\mu_{k}, \gamma$ and $\mu_{k}$. It is easy to obtain $E \left[ |x_{m,k}^2 | \right] = \sum_{k=1}^{K} \chi_{m,k}$ from (14), and by substituting (15) into (17), along with the fact of (29), the power of the received signal at $m$th L-RRH can be computed as

$$E \left[ |g_{m,k}^2 | \right] = \alpha^2 \gamma + E \left[ |\omega_{q,m,k}^2 | \right] + \alpha^2.$$  (33)

We again use the results in [11], and obtain

$$E \left[ |\omega_{q,m,k}^2 | \right] = \alpha (1 - \alpha) (\chi_m + 1).$$

By combing the above results, we complete the proof.

References