Multicell Sleeping Control and Transmit Power Adaptation in Green Heterogeneous Networks

Yi-Han Chiang* and Wanjiun Liao†
Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan*
Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan†
Email:{d99942013,wjliao}@ntu.edu.tw

Abstract—The introduction of small cells has displayed its energy-saving potentials in heterogeneous networks (HetNets) for the low operational and transmit power consumptions. To cope with the severity of inter-cell interference induced by the deployed small cells, existing research has been investigating base station (BS) sleeping incorporated with coordinated multipoint (CoMP) transmissions for the greenness of HetNets. Unfortunately, the fundamentals of multicell sleeping control and transmit power adaptation (MST) in green HetNets are in essence an NP-hard problem, which motivates us to find approximate solutions with provable performance guarantees. In this paper, we formulate the MST problem as a mixed integer linear program (MILP) and show its NP-hardness. By applying linear programming (LP) relaxation to the MST problem, we propose the progressive sleeping control with LP-based transmit power adaptation (PSLA) algorithm. We further prove that the achieved total power consumption can be upper-bounded, and show that the tightness of the upper bounds hinges on the factors of user satisfiability and network heterogeneity. Finally, the simulation results demonstrate the energy-saving performance of our proposed solution, as well as the impacts of the number of users and small cells in the network.

Index Terms—Heterogeneous networks (HetNets), coordinated multipoint (CoMP), base station (BS) sleeping, transmit power adaptation, mixed integer linear program (MILP).

I. INTRODUCTION

Heterogeneous networks (HetNets) have initiated comprehensive studies in energy related issues for cellular systems thanks to the introduction of small cells. This is because each small-cell base station (BS) is usually associated with a low operational power consumption as compared to a macrocell BS, and it can also serve user equipments (UEs) in the vicinity with a much lower transmit power. However, the exploitation of small cells may raise severe inter-cell interference and further limit the energy-saving gains, which should be addressed with precautions in green HetNets. Existing research has investigated how energy-saving techniques (such as BS sleeping [1], [2] and transmit power adjustment [3]) can be incorporated with coordinated multipoint (CoMP) transmission. In [4], Han et al. combine BS sleeping with CoMP and propose a joint transmit power and subcarrier allocation algorithm to minimize network power consumption. In [5], Hug et al. consider BS sleeping in a CoMP scenario, and propose a heuristic algorithm based on traffic thresholds to determine how many remote radio heads can be switched off. In [6], Li et al. consider how to combine BS sleeping with dynamic clustering and propose an algorithm to improve system energy efficiency. In [7], He et al. focus on the combined CoMP transmission and BS sleeping scheme and conduct a stochastic geometry analysis to evaluate energy efficiency. In [8], Wang et al. aim to jointly optimize BS sleeping, user association, and cooperative beamforming to minimize network power consumption, and develop low-complexity heuristic algorithms to solve them efficiently. The above works explore the energy-saving potentials of BS sleeping incorporated with CoMP transmission, but the essence of the multicell sleeping control and transmit power adaptation in green HetNets, which is as hard as the well-known minimum cover problem, is not yet thoroughly investigated.

In this paper, we address the fundamentals of multicell sleeping control and transmit power adaptation (MST) in green HetNets, and propose approximate solutions with bounded performance. We formulate the MST problem as a mixed integer linear program (MILP), the objective of which is to determine 1) the activeness of transmission points (TPs) and 2) the corresponding transmit power adaptation, so that the total power consumption can be minimized. Due to the NP-hardness of the original problem, we apply the linear programming (LP) relaxation to the MST problem, based on which we design the progressive sleeping control with LP-based transmit power adaptation (PSLA) algorithm to seek for an approximate solution. We also prove that the achieved total power consumption can be upper-bounded, and the tightness of the upper bounds is determined by two factors (user satisfiability and network heterogeneity), which may guide system designers to take into account for green HetNets. Our simulation results demonstrate that PSLA performs near-optimally in terms of the total power consumption, and also show the impacts of the number of UEs and TPs.

The rest of this paper is organized as follows. In Sec. II, we introduce the operations of multicell sleeping control and transmit power adaptation in green HetNets, and the adopted power consumption model. In Sec. III, we formulate the MST problem and show its NP-hardness. In Sec. IV, we design the PSLA algorithm for the relaxed MST problem, and derive upper bounds for the achieved total power consumption. In Sec. V, we show the simulation results. Finally, this paper is concluded in Sec. VI.

*In part of our prior works [9], [10], we have addressed the multicell sleeping control problem under the assumption of transmit power preallocation. In this paper, we extend our previous studies by taking transmit power adaptation into consideration.
II. SYSTEM MODEL
A. Multicell Sleeping Control and Transmit Power Adaptation in Heterogeneous Networks

In this paper, we consider the downlink of a heterogeneous network composed of a set $\mathcal{M}$ of TPs (e.g., a macro cell and multiple spatially and spectrally coexisting small cells) and a set $\mathcal{N}$ of UEs (see Fig. 1). Universal frequency reuse is adopted by all TPs to fully utilize the network spectrum.

Our aim is to enable TPs to jointly perform sleeping control and transmit power adaptation in green HetNets. For the sleeping control, each TP can be activated to serve UEs, or deactivated to reduce its operational power consumption. Typically, this simple operation deactivating some hardware components can save energy substantially since a great portion of energy dissipation is attributed to operating in active mode (see [11]). Therefore, a reduced operational power consumption results whenever BS sleeping takes place.

For the transmit power adaptation, multiple TPs can adjust their transmit powers and jointly transmit to UEs. Such an adaptation can use transmit power budgets flexibly, while improving UEs’ received signal qualities. Denote by $\text{SINR}_j$ the received signal-to-interference-plus-noise ratio (SINR) at UE $j \in \mathcal{N}$, which can be expressed as

$$\text{SINR}_j = \frac{\sum_{i \in \mathcal{M}} p_{ij} |h_{ij}|^2}{\sum_{i \in \mathcal{M}} \sum_{k \in \mathcal{N}\setminus\{j\}} p_{ik} |h_{ij}|^2 + \sigma^2},$$

where $|h_{ij}|^2$ is the channel gain from TP $i$ to UE $j$, and $\sigma^2$ is the noise spectral density at each UE. The transmit power $p_{ij}$ from TP $i$ to UE $j$ satisfies $\sum_{j \in \mathcal{N}} p_{ij} \leq p_i^{\text{max}}$, where $p_i^{\text{max}}$ is the RF output power of TP $i$ at its maximum load, namely TP $i$’s transmit power budget.

B. Power Consumption Model

The total power consumption is to aggregate the operational and transmit power consumptions of all TPs. Denote by $p^{\text{tot}}$ the total power consumption, which is given by

$$p^{\text{tot}} = \sum_{i \in \mathcal{M}} p_i^{\text{act}} y_i + \sum_{i \in \mathcal{M}} p_i^{\text{slp}} (1 - y_i) + \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} \Delta_i p_{ij},$$

where $p_i^{\text{act}}$, $p_i^{\text{slp}}$, and $p^{\text{tot}}$ represent the power consumptions of sleeping control, transmit power adaptation, and the constant part of $p^{\text{tot}}$, respectively. The parameter $p_i^{\text{act}} = p_i^{\text{act}} - p_i^{\text{slp}}$, where $p_i^{\text{act}}$ and $p_i^{\text{slp}}$ are the power consumptions of TP $i$ operating in active mode and sleep mode, respectively. The binary variable $y_i$ indicates the activeness of TP $i$, which is 1 if TP $i$ is in active mode, and 0 in sleep mode, and $\Delta_i$ is the slope of the load-dependent power consumption of TP $i$.

III. PROBLEM FORMULATION

Mathematically, the MST problem can be formulated as

$$\mathbb{P}_{\text{MST}} : \min_{y,p} p^{\text{tot}},$$

s.t. \hspace{1cm} \text{SINR}_j \geq \text{SINR}_0, \quad \forall j \in \mathcal{N}, \quad (3a)$$
$$\sum_{j \in \mathcal{N}} p_{ij} \leq y_i p_i^{\text{max}}, \quad \forall i \in \mathcal{M}, \quad (3b)$$
$$y_i \in \{0, 1\}, \quad \forall i \in \mathcal{M}, \quad (3c)$$
$$p_{ij} \geq 0, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}. \quad (3d)$$

The constraints (3a) states that the received SINR should meet the SINR threshold $\text{SINR}_0$ for each UE, (3b) indicates that each TP can allocate transmit powers to UEs (up to its transmit power budget) only if it is operating in active mode. The constraints (3c) and (3d) refer to the decision variables of sleeping control and transmit power adaptation, respectively.

The goal of the MST problem is to identify a set of active TPs and the corresponding transmit power adaptation so that the total power consumption is minimized, while the constraints (3a), (3b), (3c), and (3d) can all be met. However, such a problem is NP-hard (as proved in Theorem 1), and hence we are motivated to find approximate solutions for the MST problem with provable bounds.

**Definition 1.** (Polynomial-Time Reducibility [12, p. 466]) For any two decision problems $\mathcal{A}$ and $\mathcal{B}$, we write $\mathcal{A} \preceq_{\text{P}} \mathcal{B}$ if $\mathcal{A}$ can be reduced to $\mathcal{B}$ in polynomial time.

**Definition 2.** (MULTICELL SLEEPING CONTROL AND TRANSMIT POWER ADAPTATION PROBLEM) Consider the MST problem $\mathbb{P}_{\text{MST}}$. Does there exist an activeness $y$ of TPs and transmit powers $p$ while satisfying the constraints (3a), (3b), (3c), and (3d), such that the total power consumption $p^{\text{tot}}$ is at most $U$? We describe the problem instance as

$$\text{MST} \left( \mathcal{M}, \mathcal{N}, \left\{ p_i^{\text{act}} \right\}_i, \left\{ p_i^{\text{slp}} \right\}_i, \left\{ \Delta_i \right\}_i, \left\{ |h_{ij}|^2 \right\}_i, \right), \sigma^2, \text{SINR}_0, \left\{ p_i^{\text{max}} \right\}_i, U \right).$$

**Definition 3.** (MINIMUM COVER (MC) PROBLEM) Given a collection $\mathcal{C}$ of subsets of a finite set $\mathcal{S}$ with $\bigcup_{C \in \mathcal{C}} C = \mathcal{S}$ and a positive integer $V$. Does $\mathcal{C}$ contain a cover for $\mathcal{S}$, i.e., a subcollection $C' \subseteq \mathcal{C}$ with $|C'| \leq V$, such that every element of $\mathcal{S}$ belongs to at least one member of $C'$? The problem instance can be denoted by $\text{MC}(\mathcal{S}, \mathcal{C}, V)$.

**Theorem 1.** MINIMUM COVER PROBLEM $\preceq_{\text{P}}$ MULTICELL SLEEPING CONTROL AND TRANSMIT POWER ADAPTATION PROBLEM.
Proof: We prove the NP-hardness of the MST problem by restriction. In particular, we restrict the MST problem by allowing only instances with simply on-off power consumptions ($p^\text{on}_i = 1$, $p^\text{off}_i = 0$, $\Delta_i = 0$), binary channel states ($|h_{ij}|^2 \in \{0, 1\}$), a certain noise power ($\sigma^2 = \min_{i \in \mathcal{N}} \sum_{j \in \mathcal{M}} |h_{ij}|^2 / |\mathcal{N}|$), and equal transmit power budgets ($p^\text{max}_i = 1$). Notice that the MST problem involves transmit power adaptation whereas the MC problem does not have an analogus part, we have to make the SINR threshold sufficiently low so as to ensure the SINR-feasibility for all UEs. According to (1), the constraint (3a) can be reinterpreted as, for each UE $j \in \mathcal{N}$,

$$\text{SINR}_0 \leq \left( \frac{\sum_{i \in \mathcal{M}} \sum_{k \in \mathcal{N}} p_{ik} |h_{ij}|^2 + \sigma^2}{\sum_{i \in \mathcal{M}} p_{ij} |h_{ij}|^2 - 1} \right)^{-1}. \quad (4)$$

By setting $\text{SINR}_0$ to $1/|\mathcal{N}|$, the restricted MST problem instances become SINR-feasible as long as each UE can be served by an active TP (each serving TP needs only allocate transmit powers uniformly, namely $p_{ij} = 1/|\mathcal{N}|$).

Now, we consider an MC problem instance of

$$\text{MC}(\mathcal{S}, \mathcal{C}, V).$$

Then, we have the corresponding restricted MST problem instance as

$$\text{MST} \left( \mathcal{C}, \mathcal{S}, \{1, \ldots, 1\}, \{0, \ldots, 0\}, \{0, \ldots, 0\}, \{\mathcal{E}_{\mathcal{C}}\}, \left\{ \frac{\min_{x \in \mathcal{S}} \sum_{x \in \mathcal{C}} \mathcal{E}_{\mathcal{C}}}{|\mathcal{S}|} \right\}, \left\{ \frac{1}{|\mathcal{S}|}, \{1, \ldots, 1\}, V \right\} \right),$$

where $\mathcal{E}_{\mathcal{C}}$ is $\mathbb{1}_{x \in \mathcal{C}}$ and $\mathbb{1}_x$ is an indicator (which is 1 if $x$ is true, and 0 otherwise). Conversely, given a restricted MST problem instance as

$$\text{MST} \left( \mathcal{M}, \mathcal{N}, \{1, \ldots, 1\}, \{0, \ldots, 0\}, \{0, \ldots, 0\}, \{\mathcal{E}_{\mathcal{M}}\}, \left\{ \frac{\min_{j \in \mathcal{N}} \sum_{i \in \mathcal{M}} |h_{ij}|^2}{|\mathcal{N}|} \right\}, \left\{ \frac{1}{|\mathcal{N}|}, \{1, \ldots, 1\}, U \right\} \right),$$

Then, we have the corresponding MC problem instance of

$$\text{MC}(\mathcal{N}, \{\mathcal{J}_i\}, U),$$

where $\mathcal{J}_i = \{j \mid |h_{ij}|^2 = 1, j \in \mathcal{N}\}$. Since the transformation between the restricted MST problem instance and the MC problem instance is a one-to-one mapping and it can be run in polynomial time, it is thus proved that the MST problem is NP-hard.  

IV. ALGORITHM DESIGN AND ANALYSIS

A. Algorithm Design

Our design principle relies on the LP relaxation, which is to relax the discrete nature of the constraint (3c) and then transform the MST problem to a linear program. In particular, we have the relaxed MST problem as

$$
\forall \mathcal{M}, \mathcal{N}, \{p^\text{on}_i\}, \{p^\text{off}_i\}, \{\Delta_i\}, \{|h_{ij}|^2\}, \sigma^2, \text{SINR}_0, \{p^\text{max}_i\}, \quad (5a)
$$

$$\hat{p}_{ij} \in \{0, 1\}, \quad \forall (i, j) \in \mathcal{M} \times \mathcal{N}.
$$

**Algorithm 1** Progressive Sleeping Control with LP-Based Transmit Power Adaptation (PSLA) Algorithm

**Input:** $\mathcal{M}, \mathcal{N}, \{p^\text{on}_i\}, \{p^\text{off}_i\}, \{\Delta_i\}, \{|h_{ij}|^2\}, \sigma^2, \text{SINR}_0, \{p^\text{max}_i\}$

**Output:** $y^1, p^1$

1. Set $y' \leftarrow \{y'_i = 1, i \in \mathcal{M}\}$
2. Sort $\mathcal{M}$ into $\mathcal{M}'$ in descending order of $\{p^\text{on}_i\}$
3. for each $m \in \mathcal{M}'$ do
   4. Let $i^m$ be the $m$-th TP of $\mathcal{M}'$
   5. Set $y'' \leftarrow y'$ and $y''_i \leftarrow 0$
   6. Solve $\mathcal{P}_{\mathcal{L}}^\mathcal{M}(p|y'')$ to obtain $p''$
   7. if $p''$ is feasible then
      8. Set $y''_i \leftarrow 0$
   9. Set $y^1 \leftarrow y''$ and $p^1 \leftarrow p''$

Our designed PSLA algorithm (see Algorithm 1) aims to determine an activeness $y^1$ of TPs and transmit power adaptation $p^1$. We begin with all TPs in active mode and see how many of them can be deactivated. First, we sort TPs in descending order of their operational powers $\{p^\text{on}_i\}$. Then, we try to deactivate one TP at a time according to the above ordering. If such a deactivation (the resulting activeness is denoted by $y''$) retains the feasibility of $\mathcal{P}_{\mathcal{L}}^\mathcal{M}(p|y'')$, the TP can be deactivated safely. After scanning through all TPs, the algorithm terminates with a suboptimal solution $(y^1, p^1)$.

The PSLA algorithm takes $O(|\mathcal{M}| \log |\mathcal{M}|)$ operations for sorting TPs and solves the relaxed MST problem $|\mathcal{M}|$ times. According to the Karmarkar’s algorithm [13], each time of solving the LP program requires $O\left(|\mathcal{M}|^{3.5} L^2\right)$, where $L$ is the number of bits in the input. Altogether, we have the time complexity of the PSLA algorithm as $O\left(|\mathcal{M}|^{4.5} L^2\right)$.

B. Bounding Analysis

Let us denote by $O^*$, $O^\mathcal{L}$, $O^1$ the total power consumptions of an optimal solution $(y^*, p^*)$ to $\mathcal{P}_{\mathcal{M}}^\mathcal{L}$, an optimal LP solution $(y^\mathcal{L}, p^\mathcal{L})$ to $\mathcal{P}_{\mathcal{L}}^\mathcal{M}$, and a suboptimal solution $(y^1, p^1)$ achieved by the PSLA algorithm, respectively, which can be expressed as (for brevity, $(\cdot) \in \{*, \mathcal{L}, \mathcal{L}^1\}$)

$$
O^* = \sum_{i \in \mathcal{M}} p^\text{on}_i y_i^\text{on} + \sum_{i \in \mathcal{M}, j \in \mathcal{N}} \Delta_i p_{ij}^\text{on} + p^\text{ext},
$$

$$
= O^\mathcal{L} = O^\mathcal{L}^1,
$$

where $(\cdot) \in \{*, \mathcal{L}, \mathcal{L}^1\}$. Before reaching the upper bound of total power consumption, we first derive a lower bound for each SINR-feasible transmit power adaptation in Lemma 1. Let us sort $\mathcal{M}$ into $\mathcal{M}_j$ in descending order of $\{ |h_{ij}|^2 \}$, and denote by $\mathcal{M}_j^{(l)}$ the $l$-th TP of $\mathcal{M}_j$. Then, for each UE $j \in \mathcal{N}$, we define

$$
\hat{f}^\text{on}_j (p^{\text{cum}}) = \sum_{l=1}^{\mathcal{M}_j} \frac{|h_{M_j^{(l)} j}|^2}{\min \left\{ p^{\text{cum}}, p^\text{max}_{M_j^{(l)}} \right\} - p^\text{max}_{M_j^{(l-1)}} + p^\text{on}_{M_j^{(l)}}}.
$$

where $[z]^+ = \max (z, 0)$ and we let $p^\text{max}_{M_j^{(0)}} = 0$.
Lemma 1. For any SINR-feasible allocated transmit powers $p^o = \{p^{o}_{ij}\}$, its power consumption is lower-bounded by

$$
\sum_{j\in\mathcal{N}} \bar{p}_{j}^{\text{cum}} \leq \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} p_{ij}^{o},
$$

where $\bar{p}_{j}^{\text{cum}}$ is chosen to satisfy $f_{j}^{\text{lb}}(\bar{p}_{j}^{\text{cum}}) = \sigma^2\text{SINR}_0$.

\[\text{Proof:}\] See Appendix A.

Now, we are ready to prove that the power consumptions of transmit power adaptation and sleeping control can both be upper-bounded (see Lemmas 2 and 3), thereby obtaining the upper bound of total power consumption in Theorem 2.

Definition 4. We denote by $\lambda^{\text{US}}$ the user satisfiability as

$$
\lambda^{\text{US}} = \frac{\sum_{i\in\mathcal{M}} P_{i}^{\text{max}}}{\sum_{j\in\mathcal{N}} \bar{p}_{j}^{\text{cum}}},
$$

Lemma 2. The power consumption of transmit power adaptation can be upper-bounded by

$$
O_{\text{tpa}}^{\dagger} \leq \lambda^{\text{US}} \left( \frac{\max_{i\in\mathcal{M}} \Delta_i}{\min_{i\in\mathcal{M}} \Delta_i} \right) O_{\text{tpa}}^{\text{LP}},
$$

\[\text{Proof:}\] See Appendix B.

Lemma 3. The power consumption of sleeping control can be upper-bounded by

$$
O_{\text{sc}}^{\dagger} \leq \lambda^{\text{US}} \min \left( \left( \frac{\max_{i\in\mathcal{M}} \Omega_i}{\min_{i\in\mathcal{M}} \Omega_i} \right) O_{\text{sc}}^{\text{LP}}, \left( \frac{\max_{i\in\mathcal{M}} \Omega_i}{\min_{i\in\mathcal{M}} \Delta_i} \right) O_{\text{tpa}}^{\text{LP}} \right),
$$

where $\Omega_i = p^*/p_i^{\text{max}}, \forall i \in \mathcal{M}$.

\[\text{Proof:}\] See Appendix C.

Definition 5. We denote by $\lambda^{\text{NH}}$ the network heterogeneity as

$$
\lambda^{\text{NH}} = \min \left( \lambda^{\text{NH-1}}, \lambda^{\text{NH-2}} \right),
$$

where

$$
\lambda^{\text{NH-1}} = \max \left( \frac{\max_{i\in\mathcal{M}} \Omega_i}{\min_{i\in\mathcal{M}} \Omega_i}, \frac{\max_{i\in\mathcal{M}} \Delta_i}{\min_{i\in\mathcal{M}} \Delta_i} \right),
$$

$$
\lambda^{\text{NH-2}} = \frac{\max_{i\in\mathcal{M}} \Omega_i + \max_{i\in\mathcal{M}} \Delta_i}{\min_{i\in\mathcal{M}} \Delta_i}.
$$

Theorem 2. The total power consumption can be upper-bounded by

$$
O^{\dagger} \leq \lambda^{\text{US}} \lambda^{\text{NH}} O^*. 
$$

\[\text{Proof:}\] With the results of Lemmas 2 and 3, we obtain

$$
O_{\text{sc}}^{\dagger} + O_{\text{tpa}}^{\dagger} \leq \lambda^{\text{US}} \min \left( \frac{\max_{i\in\mathcal{M}} \Omega_i}{\min_{i\in\mathcal{M}} \Omega_i} O_{\text{sc}}^{\text{LP}}, \frac{\max_{i\in\mathcal{M}} \Delta_i}{\min_{i\in\mathcal{M}} \Delta_i} O_{\text{tpa}}^{\text{LP}} \right) \left( \frac{\max_{i\in\mathcal{M}} \Omega_i}{\min_{i\in\mathcal{M}} \Delta_i} + \frac{\max_{i\in\mathcal{M}} \Delta_i}{\min_{i\in\mathcal{M}} \Delta_i} \right). \quad (8)
$$

By using the definitions of $O^{\dagger}$ and $O^{\text{LP}}$, as well as the facts that $\lambda^{\text{NH-1}} \geq 1$ and $\lambda^{\text{NH-2}} \geq 1$, we get

$$
O^{\dagger} = O_{\text{sc}}^{\dagger} + O_{\text{tpa}}^{\dagger} + \hat{p}^{\text{est}} \leq \lambda^{\text{US}} \min \left( \lambda^{\text{NH-1}}, \lambda^{\text{NH-2}} \right) O^{\text{LP}}, \quad (9)
$$

which completes the proof according to Definition 5 and the natural fact that $O^{\text{LP}} \leq O^*$.

It can be observed from Theorem 2 that the tightness of the upper bounds hinges on two key factors:

- For the user satisfiability, the easier the UEs’ satisfaction in their received SINRs, the greater the gap between transmit power budgets and the cumulative allocated transmit powers (see Fig. 3), and thus the higher the value of $\lambda^{\text{US}}$.
- For the network heterogeneity, the more heterogeneous the network deployment, the more different the power consumption profile among TPs (see TABLE I), and hence the greater the value of $\lambda^{\text{NH}}$.

These two factors are jointly affecting the tightness of the upper bounds, which suggests, for future system designers, to take them into account while performing multicell sleeping control and transmit power adaptation, whenever a tighter upper bound for the total power consumption is desired.

V. PERFORMANCE EVALUATION

We consider a HetNet consisting of a macro cell and multiple pico cells deployed within the macro-cell coverage. Multicell sleeping control and transmit power adaptation is performed among all TPs, and our simulation results are averaged over 1000 iterations. The other settings for our simulation are summarized in TABLE I and II, and more detailed descriptions can be found in [14], [15].

To demonstrate the energy-saving performance of our proposed solution, we compare the following approaches:

i) OPT-LP: Solve $\mathbb{P}_{\text{MST}}(y, p)$ to obtain fractional $y$ and $p$ that minimize $p^{\text{tot}}$.

ii) OPT-MILP: Exhaustively search for an integral $y$ and obtain corresponding $p$ via $\mathbb{P}_{\text{MST}}(p|y)$ to minimize $p^{\text{tot}}$.

iii) PS-Only: Progressively deactivate TPs as in Algorithm 1, except that $p$ is forced to $\{p_{ij} = p^{\text{max}}/|\mathcal{N}|\}$.

iv) LA-Only: Force $y = \{1, \ldots, 1\}$ and solve $\mathbb{P}_{\text{MST}}(y|p)$ to obtain corresponding $p$ that minimizes $p^{\text{tot}}$.

v) MAX: Force $y = \{1, \ldots, 1\}$ and $p = \{p_{ij} = p^{\text{max}}/|\mathcal{N}|\}$.

TABLE I: Power consumptions of various TP types [14].

<table>
<thead>
<tr>
<th>TP Type</th>
<th>$p_{i}^{\text{act}}$ [W]</th>
<th>$p_{i}^{\text{lp}}$ [W]</th>
<th>$p_{i}^{\text{max}}$ [W]</th>
<th>$\Delta_i$</th>
<th>$\Omega_i = p_{i}^{\max}/p_{i}^{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macro</td>
<td>130.0</td>
<td>75.0</td>
<td>20</td>
<td>4.7</td>
<td>2.75</td>
</tr>
<tr>
<td>Micro</td>
<td>56.0</td>
<td>39.0</td>
<td>6.3</td>
<td>2.6</td>
<td>2.70</td>
</tr>
<tr>
<td>Pico</td>
<td>6.8</td>
<td>4.3</td>
<td>0.13</td>
<td>4.0</td>
<td>19.23</td>
</tr>
<tr>
<td>Femto</td>
<td>4.8</td>
<td>2.9</td>
<td>0.05</td>
<td>8.0</td>
<td>38.00</td>
</tr>
</tbody>
</table>

Note that we solve $\mathbb{P}_{\text{MST}}$ via the Simplex Method [16]. In case no feasible solutions to $\mathbb{P}_{\text{MST}}$ exist, we simply associate the resulting power consumption with the peak value (i.e., $\sum_{i\in\mathcal{M}} p_{i}^{\text{act}} + \Delta_i p_{i}^{\text{max}}$) for all approaches for a fair comparison.

Our proposed solution is shown to outperform PS-Only, LA-Only, and MAX in terms of total power consumption (see Fig. 2a). Though PS-Only performs well in low SINR thresholds, it can be as poor as MAX in moderate and high
SINR thresholds since its uniform transmit power allocations fail to provide satisfactory SINR values to UEs. On the other hand, LA-Only works properly at all SINR thresholds, but it loses great energy-saving potentials of deactivating TPs. Our proposed solution overcomes the above problems in wisely adapting transmit powers while progressively deactivating TPs and thus acting near-optimally (w.r.t. OPT-MILP).

The numbers of UEs and TPs also have influences on the greenness of HetNets. In Fig. 2b, we see that, the total power saving of all approaches decreases with the increasing number of UEs and the decreasing number of TPs, PSLA retains its near-optimality versus affected by the increasing number of UEs and the decreasing number of TPs. Although the SINR-feasibility can be adversely affected by the increasing number of UEs and the decreasing number of TPs, PSLA retains its near-optimality for the greenness of HetNets.

VI. CONCLUSION

Multicell sleeping control and transmit power adaptation play a vital role in energy saving for HetNets. In this paper, we formulate the MST problem as an MILP, and prove its NP-hardness. We propose the PSLA algorithm to progressively deactivate TPs and adapt transmit powers via solving the relaxed MST problem through LP. We prove that the achieved total power consumption can be upper-bounded, and the tightness of the upper bounds is shown to hinge on the factors of user satisfiability and network heterogeneity. Our simulation results show that PSLA performs near-optimally due to wisely adapting transmit powers while progressively deactivating TPs. Although the SINR-feasibility can be adversely affected by the increasing number of UEs and the decreasing number of TPs, PSLA retains its near-optimality for the greenness of HetNets.

APPENDIX A

PROOF OF LEMMA 1

The allocated transmit powers are rather involved due to (1). In addition, the lower bound for $p^*$ is not attainable without actually solving all problem instances of $p^*_MST$. Therefore, we seek for a least-power cluster for each individual UE, and show that these least-power clusters result in the minimum allocated transmit powers. For this, we make the following assumptions:

- Consider that each UE is free of interference. Therefore, the constraint (3a) can be simplified to

$$\sum_{i \in M} p^*_i |h_{ij}|^2 \geq \sigma^2 \text{SINR}_0, \ \forall j \in N.$$  (10)

- Consider that all TPs are active. Each TP $i$ can allocate transmit power to a UE up to $p^*_i$, even though the sum of its allocated transmit powers may exceed $p^*_i$. In this way, the effect of the constraint (3b) is altered to

$$p^*_i \leq p^*_i, \ \forall i \in M, j \in N.$$  (11)
By combining the results of (13) and (14), we have
\[ \sum_{i \in M} \sum_{j \in N} \Delta_i \beta_{ij} \leq \left( \max_{i \in M} \Delta_i \right) \sum_{i \in M} \sum_{j \in N} \beta_{ij}. \] 
(13)
Based on Definition 4 and the result of Lemma 1, we obtain
\[ \sum_{i \in M} \sum_{j \in N} \beta_{ij} \leq \lambda_\text{US} \sum_{i \in M} \sum_{j \in N} \beta_{ij}^{\text{LP}}. \] 
(14)
By combining the results of (13) and (14), we have
\[ O_{\text{tpa}}^{\dagger} \leq \lambda_\text{US} \left( \max_{i \in M} \Delta_i \right) \sum_{i \in M} \sum_{j \in N} \beta_{ij}^{\text{LP}} \leq \lambda_\text{US} \left( \max_{i \in M} \Delta_i \frac{\max_{i \in M} \Omega_i}{\min_{i \in M} \Delta_i} \right) \sum_{i \in M} \sum_{j \in N} \Delta_i \beta_{ij}^{\text{LP}}. \] 
(15)
Then, using the definition of \( O_{\text{tpa}}^{\dagger} \) completes the proof.

**APPENDIX C PROOF OF LEMMA 3**

By the definitions of \( O_{\text{sc}}^{\dagger} \) and \( \Omega_i \), we have
\[ O_{\text{sc}}^{\dagger} = \sum_{i \in M} \beta_{i}^{\text{sc}} y_i^{\dagger} \leq \left( \max_{i \in M} \Delta_i \right) \sum_{i \in M} \beta_{i}^{\text{sc}} y_i^{\dagger}. \] 
(16)
According to the result of Lemma 1, it can be derived that
\[ \sum_{i \in M} \beta_{i}^{\text{sc}} y_i^{\dagger} \leq \sum_{i \in M} \beta_{i}^{\text{sc}} \leq \lambda_\text{US} \sum_{i \in M} \sum_{j \in N} \beta_{ij}^{\text{LP}}. \] 
(17)
Combining the results of (16) and (17) gives rise to
\[ O_{\text{sc}}^{\dagger} \leq \lambda_\text{US} \left( \max_{i \in M} \Omega_i \right) \sum_{i \in M} \beta_{i}^{\text{sc}} y_i^{\dagger} \leq \lambda_\text{US} \left( \max_{i \in M} \Omega_i \frac{\max_{i \in M} \Omega_i}{\min_{i \in M} \Delta_i} \right) \sum_{i \in M} \sum_{j \in N} \Delta_i \beta_{ij}^{\text{LP}}. \] 
(18)