Cooperative Multi-Subarray Beam Training in Millimeter Wave Communication Systems

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Abstract—This paper studies beam training design for a codebook-based beamforming millimeter wave (mmwave) system where multiple antenna arrays are employed and each array is capable of beamforming independently. To reduce the training overhead and the complexity of subsequent beam direction search, we propose a cooperative multi-subarray beam training method. Specifically, from the perspective of excluding non-effective beam direction combinations and thus reducing search space, method and criterion of beam superposition are proposed to construct a wide beam from multiple narrow beams corresponding to multiple subarrays. Then, a cooperative multi-subarray beam training scheme is proposed based on the proposed criterion. Finally, simulation results show that the proposed scheme achieves a spectral efficiency close to that of the optimal exhaustive search scheme, while has greatly reduced training overhead and computational complexity.

Index Terms—Millimeter wave communication, Multiple antenna subarray, Beam training, Beam codebook.

I. INTRODUCTION

Millimeter wave (mmwave) communication operating in the band of 30-300 GHz has gained increasing attention over the last few years [1]–[3]. It promises to provide Gbps data rate owing to the availability of large bandwidth. However, since the path-loss of mmwave signals is large, beamforming with antenna arrays is generally required to enlarge the coverage range [4], [5]. Fortunately, it is possible to pack a large number of antennas into a small size due to the short wavelength of mmwave signals.

In conventional MIMO systems, e.g., LTE, beamforming (or precoding) is entirely realized in the digital domain. In such systems, each antenna requires a dedicated energy-intensive radio frequency (RF) chain (including digital-to-analog converter, up converter, etc.), which is hard to realize for mmwave systems due to the prohibitive cost of a large number of RF chains and high energy consumption [6], [7]. In order to circumvent this difficulty, in indoor mmwave communications such as 60 GHz WLAN, beamforming is performed at the RF level using a set of phase shifters [8].

To obtain diversity/spatial multiplexing gains in such systems, array-of-subarrays (also referred to multi-subarray or sub-connected) architecture was studied in [6], [7], [9]. In such architecture, antenna array at the transmitter (Tx) and/or receiver (Rx) consists of multiple subarrays, where each of the subarrays is capable of beamforming independently using RF phase shifters. Essentially, each subarray plays a similar role in a virtual antenna in the sense of current MIMO systems. Therefore, for example, spatial multiplexing of K data streams requires that at least K subarrays are equipped by both the TX and the RX. Compared to fully-connected architecture [10], [11], the advantages of multiple subarrays architecture include reducing the number of required phase shifters, more energy-efficient and easier to be implemented [7].

The multi-subarray architecture has drawn considerable attention and been standardized owing to its attractive features [12]. Considering a codebook-based beamforming system, the feasibility of employing multiple antenna arrays is investigated in [9]. Moreover, reduced complexity algorithms for optimizing the choice of beamforming directions were developed by exploiting the sparsity of mmwave channels. Assuming perfect channel state information (CSI) at the Tx, an energy-efficient hybrid precoding algorithm with subarray architecture based on the successive interference cancelation (SIC) technique was proposed in [7] to achieve near-optimal performance.

CSI plays a key role in designing or optimizing the precoder (both baseband and RF). However, the acquisition of CSI is more complicated in mmwave systems since we cannot extract the signal received at each antenna. A viable method to overcome this difficulty is using beam training to select the preferred transmit beams and receive beams from predefined beam codebooks, by which the array gain is fully exploited and further precoding design is optimized in the reduced-size beam domain. For the case of single antenna array, a tree-based search is usually adopted to reduce the training overhead and the key issue lies in the design of hierarchical multi-resolution codebook [13], [14]. By jointly exploiting subarray and antenna deactivation techniques, a hierarchical codebook generated by closed-form expressions was devised in [15]. A heuristic approach was proposed in [16] to design a hierarchical codebook exploiting beam widening with the multi RF-chain subarray technique.

Though different criteria and methods have been proposed to design beam training codebook for single antenna array, methods of beam training and codebook design for multi-subarray architecture are seldom. It is worth to note that though some subarray techniques were proposed in [15], [16] to design training codebooks, the techniques adopted there are in fact a kind of subarray split techniques, i.e., splitting a single array into multiple subarrays. These methods cannot be directly extended to the case where multiple subarrays are
originally independent.

In this paper, to reduce the training overhead and the complexity of beam direction search, we propose a cooperative multi-subarray beam training method which also applies to the case of multiple independent subarrays. Specially, to speed up the process of beam training, the idea of beam superposition is proposed to construct a wide beam from the narrow beams corresponding to multiple subarrays. A criterion of beam superposition is further proposed to reduce the mutual interference caused by different independent subarrays. Then, we propose a cooperative multi-subarray beam training method to sweep the entire beam space using the constructed wide beams. By excluding non-effective beam direction combinations (BDCs), the space of BDC can be greatly reduced. Finally, simulation results show that compared with the optimal but computationally prohibitive exhaustive search scheme, the proposed scheme suffers a very small performance loss in terms of spectral efficiency (SE), while has significantly reduced training overhead and much lower computational complexity.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a mmwave communication system, where the Tx and Rx are equipped with \( L_T \) and \( L_R \) subarrays, respectively. The \( i \)-th transmitting subarray and the \( j \)-th receiving subarray are equipped with \( N_{T,i} \) and \( N_{R,j} \) antennas, respectively. For simplicity, it is assumed that \( N_{T,1} = \cdots = N_{T,L_T} = N_{T,0} \). Therefore, the total number of antennas employed at the Tx is \( N_T = L_T N_{T,0} \). Similarly, the total number of antennas employed at the Rx is \( N_R = L_R N_{R,0} \), where \( N_{R,0} \) is the number of antennas of each receiving subarray. The architectures of system and multi-subarray are depicted in Fig.1.

For simplicity, we assume that each subarray employed at the Tx (Rx) is a uniform linear array (ULA). The \( L_T \) (\( L_R \)) transmitting (receiving) subarrays form a bigger linear antenna array. \( \mathbf{a}_T(x) = [\mathbf{a}_{T,1}^H(x), \cdots, \mathbf{a}_{T,L_T}^H(x)]^H \) collects \( L_T \) array response vectors of \( L_T \) transmitting subarrays, where \( \mathbf{a}_{T,i}(x) \) is the array response vector of the \( i \)-th subarray. \( \mathbf{a}_T(x) \) is given by

\[
\mathbf{a}_T(x) = \frac{1}{\sqrt{N_T}} \begin{bmatrix} 1, e^{j2\pi d_1 x}, \cdots, e^{j2\pi d_{N_{T,0}} x} \end{bmatrix}.
\]

where \( \lambda \) and \( d_k (2 \leq k \leq N_T) \) are the signal wavelength and the distance between the \( k \)-th antenna and the 1-th antenna at the Tx, respectively. Apparently, \( \mathbf{a}_{T,1} \) takes the form

\[
\mathbf{a}_{T,1}(x) = \frac{1}{\sqrt{N_T}} \begin{bmatrix} 1, e^{j2\pi d_1 x}, \cdots, e^{j2\pi d_{(N_{T,0}-1)} x} \end{bmatrix}.
\]

For the Rx, \( \mathbf{a}_R \) and \( \mathbf{a}_{R,j} \) taking similar forms denote the array response vectors of entire array and \( j \)-th subarray, respectively.

In this paper, we adopt an extended Saleh-Valenzuela geometric model to model propagation in the mmwave regime [5], [13]. The channel matrix is given by

\[
\mathbf{H} = \sqrt{N_T N_R / \beta} \sum_{l=1}^{L_p} \alpha_l \mathbf{a}_R(\Omega_l) \mathbf{a}_T^H(\Phi_l),
\]

where \( \alpha_l \sim \mathcal{CN}(0, \sigma_n^2) \) is the complex gain of the \( l \)-th path, \( \beta \) is the average path-loss, and \( L_p \) is the number of paths. In (3), \( \Omega_l = \cos(\theta_l) \) and \( \Phi_l = \cos(\psi_l) \), where \( \theta_l \) and \( \psi_l \) are the physical AoA and AoD of the \( l \)-th path, respectively.

Let \( x \in \mathbb{C}^{L_T \times 1} \) denote the transmitted signal, the signal received at the Rx is given by

\[
\mathbf{y} = \mathbf{W}_{RF}^H \mathbf{H} \mathbf{F} \mathbf{R} \mathbf{F}^H \mathbf{R} \mathbf{F} \mathbf{x} + \mathbf{W}_{RF}^H \mathbf{n}.
\]

where \( \mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_R}) \) is the \( N_R \times 1 \) noise vector, and \( \mathbf{W}_{RF} \) and \( \mathbf{F} \) are the \( N_R \times L_R \) RF combiner and \( N_T \times L_T \) RF precoder, respectively. For the multi-subarray architecture, \( \mathbf{F} \) and \( \mathbf{W}_{RF} \) possess a special structure, i.e., block diagonal structure. The reason is that each subarray is connected to only one RF chain. In this case, \( \mathbf{F} \) takes the form

\[
\mathbf{F} = \text{diag}(f_1, \cdots, f_{L_T}),
\]

where \( f_i \) is the RF precoding vector for the \( i \)-th subarray (\( 1 \leq i \leq L_T \)). It is similar for \( \mathbf{W}_{RF} \). Since \( \mathbf{W}_{RF}^H \mathbf{F} = \mathbf{I}_{L_R} \) holds, the components of \( \mathbf{W}_{RF}^H \mathbf{n} \) are independently and identically distributed as \( \mathcal{CN}(0, \sigma_n^2) \). Therefore, we can equivalently write (4) as follows

\[
\mathbf{y} = \mathbf{W}_{RF}^H \mathbf{H} \mathbf{F} \mathbf{x} + \mathbf{n},
\]

where \( \mathbf{n} \) is distributed as \( \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_R}) \).

B. Problem Formulation

Similar to [9], we consider a codebook-based beamforming/combining for all subarrays. The codebook consisting of \( \mathcal{M}_T \) (\( \mathcal{M}_R \)) analog beamforming vectors used for the \( i \)-th (\( j \)-th) subarray at the Tx (Rx) is denoted by \( \mathcal{C}_{T,i}(\mathcal{C}_{R,j}) \). For simplicity, it is further assumed that \( \mathcal{C}_{T,1} = \cdots = \mathcal{C}_{T,L_T} = \mathcal{C}_{T,0} \) and \( \mathcal{C}_{R,1} = \cdots = \mathcal{C}_{R,L_R} = \mathcal{C}_{R,0} \). Each beamforming vector in \( \mathcal{C}_{T,0}(\mathcal{C}_{R,0}) \) steers the beam of transmitting (receiving) subarray towards a certain angle \( \Phi \in [-1, 1] \). The codebooks \( \mathcal{C}_{T,0} \) and \( \mathcal{C}_{R,0} \) are given by

\[
\mathcal{C}_{T,0} = \{f_1, f_2, \cdots, f_{M_T}\}, \quad \mathcal{C}_{R,0} = \{w_1, w_2, \cdots, w_{M_R}\},
\]

where \( f_i \) and \( w_j \) take the forms \( \mathbf{a}_{T,1} \) and \( \mathbf{a}_{R,1} \), respectively.
Since any subarray at the Tx can pick any codeword in $C_{T,0}$, there are totally $M_{T}^{L_{T}}$ possible RF precoding matrices, i.e., $F_{RF} \in C_{T,0}^{L_{T}}$ and $\text{card}(C_{T,0}) = M_{T}^{L_{T}}$. Note that $\text{card}(\cdot)$ denotes the cardinality of a set, i.e., number of elements in the set. It is similar for the Rx, i.e., $W_{RF} \in C_{R,0}^{L_{R}}$ and $\text{card}(C_{R,0}) = M_{R}^{L_{R}}$. Given $W_{RF}$ and $F_{RF}$, we obtain an equivalent channel matrix $\bar{H} = W_{RF}^{\dagger}HF_{RF}$. The mutual information achieved over this equivalent channel is given by

$$I(\bar{H}) = \log_{2} \det \left( I + \sigma_{n}^{-2}\bar{H}^{\dagger}\bar{H} \right).$$

The goal is to maximize $I(\cdot)$ by selecting appropriate $F_{RF}$ and $W_{RF}$. The optimization problem can be formulated as

$$\left( W_{RF}^{*}, F_{RF}^{*} \right) = \arg \max_{W_{RF}, F_{RF}} \log_{2} \det \left( I + \sigma_{n}^{-2}\bar{H}^{\dagger}\bar{H} \right) \tag{8}$$

Since an estimation of $H$ is challenging in mmwave communications due to the large number of antennas, we focus on the equivalent channel $\bar{H}$ obtained via beam training. One method to solve (8) is exhaustive search, i.e., use all possible matrix pairs to transmit and receive reference signals and then find out the best one. The estimation of all equivalent channel matrices requires the following channel measurements

$$H = \{ h_{i,j,b_{R},b_{T}} \mid 1 \leq i \leq L_{R}, 1 \leq j \leq L_{T}, 1 \leq b_{R} \leq M_{R}, 1 \leq b_{T} \leq M_{T} \}, \tag{9}$$

where $i,j,b_{R},b_{T}$ are the Rx subarray index, the Tx subarray index, the beam index picked at the Rx subarray and the beam index picked at the Tx subarray [9].

Since the size of the space of beam direction combination (BDC) is $M_{T}^{L_{T}}M_{R}^{L_{R}}$, which scales exponentially with $L_{T}$ and $L_{R}$, the beam training overhead is very large. If the space of BDC which needs to be considered can be reduced, the training overhead can be reduced accordingly. Conversely, if the training overhead is decreased, i.e., $\text{card}(\mathcal{H})$ is decreased, the computational complexity to process $\mathcal{H}$ can be reduced as well. In the following, we will propose a cooperative multi-subarray beam training method to reduce this space by exploiting the sparsity of mmwave channels.

### III. Subarray Cooperation - Beam Superposition Approach

Thanks to the sparsity of mmwave channels, though the number of BDCs is large, i.e., $M_{T}^{L_{T}}M_{R}^{L_{R}}$, lots of BDCs are in fact non-effective. To illustrate this point, we assume that both the Tx and the Rx are equipped with two subarrays. As shown in Fig.2, the BDC $(1, 1; 1, 1)$ is non-effective since it does not match with any path of the channel. Therefore, it is almost impossible to establish reliable communication links based on these BDCs. To reduce the space of BDC, we should try our best to find and exclude those non-effective BDCs using a small amount of training overhead. By excluding lots of non-effective BDCs, a reduced space constituted by the remaining BDCs can be much small. Based on the reduced space, the complexity of subsequent algorithms, e.g., beam training or beam direction search, can be greatly reduced.

To find and exclude those non-effective BDCs, we initially sweep the entire beam space with wide beams, which is similar to the sector level sweep in IEEE 802.11ad. However, for a given codebook, the beamwidth is fixed and usually narrow in order to achieve a high array gain. Note that since each subarray is capable of beamforming independently, if two adjacent narrow beams from the two subarrays at the Tx (Rx) are used simultaneously, the superimposed beam may have a large beamwidth, as shown in Fig.3-1.

![Fig. 2. An illustration of non-effective beam direction combination. The blue lines represent the paths of channel. The BDC $(1, 1; 1, 1)$ is non-effective.](image)

![Fig. 3. (1) Desirable magnitude response of independent and superimposed beams. (2) Magnitude response of independent and superimposed beams.](image)

But this may not be always the case, as shown in Fig.3-2. Although the beam superimposed by two independent beams has larger beamwidth, there is an undesirable beam nulling in the overlapping area. The reason is that there exists mutual interference between the two subarrays. Let $\Phi_{k}$ and $\Phi_{l}$ be two codewords selected from $C_{T,0}$ with center angles of main lobe $\Phi_{k}$ and $\Phi_{l}$, respectively. The beam response is calculated as

$$r(\Phi) = a_{T,1}^{H}(\Phi)f_{k} + a_{T,2}^{H}(\Phi)f_{l} \tag{10}$$

$$= \frac{1}{N_{T}} \sum_{n=0}^{N_{T}-1} e^{-j\frac{2\pi d_{a}}{\lambda}n(\Phi-\Phi_{k})} +$$

$$e^{-j\frac{2\pi d_{a}}{\lambda}(N_{T,0}+K_{s})\Phi} \sum_{n=0}^{N_{T}-1} e^{-j\frac{2\pi d_{a}}{\lambda}n(\Phi-\Phi_{l})} \tag{11}$$

where $K_{s}$ is the ratio between $d_{s}$ and $d_{c}$ (See Fig.1-2 for $d_{s}$ and $d_{c}$). Eq.(11) indicates that the distance between the two
subarrays, in fact, has an impact on the magnitude response of the superimposed beam. In general, the mutual interference, which implies that using adjacent narrow beams to construct a wide beam is not always viable.

For ease of analysis, for two real numbers $x_1, x_2 \in \mathbb{R}$, a “circle” distance between $x_1$ and $x_2$ is defined as

$$d(x_1, x_2) = |e^{jx_1} - e^{jx_2}|.$$  \hspace{1cm} (12)

A small $d(f_k, f_l)$ implies that $f_k$ and $f_l$ are two adjacent codewords. Then there may exist undesirable beam nulling in the overlapping area. However, if we give up using adjacent narrow beams to construct a wide beam, we can still obtain a wide beam using beam superposition method. As shown in Fig.4, logically the superimposed beam has larger beamwidth, when $f_k$ and $f_l$ are not adjacent narrow beams. In contrast to conventional wide beams, wide beams in this paper are in fact multi-beams. In general, the larger the distance $d(f_k, f_l)$ is, the smaller the possible loss of array gain in the main-lobes is. In fact, we have the following proposition.

**Proposition 1.** Let $f_k$ and $f_l$ be two beamforming vectors with center angles $\Phi_k$ and $\Phi_l$, respectively. If $d(f_k, f_l)$ is relatively large, the beam response $r(\Phi)$ in (10) satisfies

$$|r(\Phi)| \approx \begin{cases} |a_{I,1}^H(\Phi)f_k| \approx 0.5, & \Phi \in (\Phi_k - \varepsilon, \Phi_k + \varepsilon) \\ |a_{I,2}^H(\Phi)f_l| \approx 0.5, & \Phi \in (\Phi_l - \varepsilon, \Phi_l + \varepsilon) \end{cases}$$

where $\varepsilon > 0$ is a small positive real number. \hspace{1cm} (13)

The proof of Proposition 1 and subsequent Proposition 2 is omitted due to space limitation. Please refer to [17] for more details. Next we consider how to apply the beam superposition method to multiple beams and derive a beam superposition criterion.

**Definition 1.** Let $\Phi_1, \Phi_2, \ldots, \Phi_K$ be the center angles of $K$ beams. For an index set $I \subset K = \{1, \ldots, K\}$, the distance of $I$, denoted by $d(I)$, is defined as

$$d(I) = \min \{d(\Phi_i, \Phi_j) \mid i \neq j, i \in I, j \in I\}.$$ \hspace{1cm} (14)

For simplicity, we assume that $K$ can be written as a product of two positive integers $I$ and $J$, i.e., $K = IJ$.  

**Definition 2.** The set $\{I_1, \ldots, I_I\}$ is called a partition of equal cardinality (PEC) of $K$ if it is a partition of $K$ and $\text{card}(I_1) = \cdots = \text{card}(I_I)$. The distance of $\{I_1, \ldots, I_I\}$, denoted by $d(I_1, \ldots, I_I)$, is defined as

$$d(I_1, \ldots, I_I) = \min \{d(I_1), \ldots, d(I_I)\}.$$ \hspace{1cm} (15)

With a PEC $\{I_1, \ldots, I_I\}$ available, we can construct another PEC $\{J_1, \ldots, J_J\}$ of $K$ from $\{I_1, \ldots, I_I\}$, termed as dual PEC (DPEC). The procedure is as follows $(k = 1, \ldots, I)$:

$$J_1 = \{i_{1,1}, i_{1,2}, \ldots, i_{1,l}\}, \quad i_{1,k} \in I_k;$$

$$J_2 = \{i_{2,1}, i_{2,2}, \ldots, i_{2,l}\}, \quad i_{2,k} \in I_k \setminus \{i_{1,k}\};$$

$$\vdots$$

$$J_J = \{i_{J,1}, i_{J,2}, \ldots, i_{J,l}\}, \quad i_{J,k} \in I_k \setminus \{i_{1,k}, \ldots, i_{J-1,k}\}.$$  

The PEC $\{I_1, \ldots, I_I\}$ and its DPEC $\{J_1, \ldots, J_J\}$ are termed as a PEC pair, denoted by $(I_1, \ldots, I_I; J_1, \ldots, J_J)$. Now we can state the criterion of beam superposition.

**Criterion of Beam Superposition (CBS):** Let $\Phi_1, \Phi_2, \ldots, \Phi_K$ be the center angles of $K$ beams. Let $\{I_1, \ldots, I_I\}$ and $\{J_1, \ldots, J_J\}$ be a PEC pair of $\{1, \ldots, K\}$. The PEC pair $(I_1, \ldots, I_I; J_1, \ldots, J_J)$ is satisfied the criterion of beam superposition if any other PEC pair $(I_1, \ldots, I_I; K_1, \ldots, K_J)$, the following inequality holds

$$d(J_1, \ldots, J_J) \geq d(I_1, \ldots, I_I).$$ \hspace{1cm} (16)

A PEC pair satisfying the CBS is termed as a CBS-PEC pair.

For a CBS-PEC pair $(I_1, \ldots, I_I; J_1, \ldots, J_J)$, each element in $\{J_1, \ldots, J_J\}$ can be used to construct a wide beam by beam superposition method which has the minimal mutual interference in the main-lobes. The physical meaning of $\{I_1\}$ is provided in the next section. The following proposition plays a key role in cooperative multi-subarray beam training.

**Proposition 2.** Let $\Phi_1, \Phi_2, \ldots, \Phi_K$ be the center angles of $K$ beams such that $\Phi_i - \Phi_j = 2(i-j)/K, (i > j)$. A CBS-PEC pair $(I_1, \ldots, I_I; J_1, \ldots, J_J)$ is given by

$$I_i = \{(i-1)J + k \mid k = 1, 2, \ldots, J\}, (i = 1, \ldots, I) \hspace{1cm} (17)$$

$$J_j = \{(j+k-1)I \mid k = 1, 2, \ldots, I\}, (j = 1, \ldots, J). \hspace{1cm} (18)$$

**IV. COOPERATIVE MULTI-SUBARRAY BEAM TRAINING**

In this section, we propose a cooperative multi-subarray beam training method based on CBS. We first consider the case where both the Tx and Rx are equipped with two subarrays. Then we consider the general case. The codebook $C_{T,0}$ of size 12 is shared by the two subarrays at the Tx. The beam pattern of each element in $C_{T,0}$ is shown in Fig.5-1. Similarly, the codebook $C_{R,0}$ of size 8 is shared by the two subarrays at the Rx.

The beam patterns of $C_{R,0}$ are shown in Fig.5-2.

Let $I = 2$. According to Proposition 2, a CBS-PEC pair $(I_{T,1}, I_{T,2} ; J_{T,1}, \ldots, J_{T,6})$ for 12 beams in $C_{T,0}$ is given by

$I_{T,1} = \{1, 2, 3, 4, 5, 6\}, \quad I_{T,2} = \{7, 8, 9, 10, 11, 12\};$

$J_{T,1} = \{1, 7\}, \quad J_{T,2} = \{2, 8\}, \quad J_{T,3} = \{3, 9\};$

$J_{T,4} = \{4, 10\}, \quad J_{T,5} = \{5, 11\}, \quad J_{T,6} = \{6, 12\}.$
Similarly, a CBS-PEC pair \((\mathcal{I}_R, 1, \mathcal{I}_R, 2; \mathcal{J}_R, 1, \cdots, \mathcal{J}_R, 4)\) for 8 beams in \(\mathcal{C}_{R, 0}\) is given by
\[
\mathcal{I}_R, 1 = \{1, 2, 3, 4\}, \quad \mathcal{I}_R, 2 = \{5, 6, 7, 8\}; \quad \mathcal{J}_R, 1 = \{1, 5\}, \quad \mathcal{J}_R, 2 = \{2, 6\}, \quad \mathcal{J}_R, 3 = \{3, 7\}, \quad \mathcal{J}_R, 4 = \{4, 8\}.
\]

The beam space \([-1, 1]\) at the Tx is originally covered by subarray 1 (TX1) and subarray 2 (TX2) independently and repeatedly. But now, the 1-st beam to the 6-th beam (totally six beams) are affiliated to TX1. In other words, TX1 covers the beam space \([-1, 0]\), as shown by six red solid curves in Fig.5-1. The remaining beam space \([0, 1]\) is covered by TX2 with the other six beams (i.e., the 7-th beam to the 12-th beam) in \(\mathcal{C}_{R, 0}\). It is similar for the Rx. In general, for a CBS-PEC pair \((\mathcal{I}_T, 1, \cdots, \mathcal{I}_T, L_T; \mathcal{J}_T, 1, \cdots, \mathcal{J}_T, J_T)\), each element \(\mathcal{I}_T, i\) corresponds to a beam subspace covered by one subarray.

Let \(J_T = M_T/L_T\) and \(J_R = M_R/L_R\). Let the CBS-PEC pairs of Tx and Rx be \((\mathcal{I}_T, 1, \cdots, \mathcal{I}_T, L_T; \mathcal{J}_T, 1, \cdots, \mathcal{J}_T, J_T)\) and \((\mathcal{I}_R, 1, \cdots, \mathcal{I}_R, L_R; \mathcal{J}_R, 1, \cdots, \mathcal{J}_R, J_R)\). The cooperative beam training process is as follows. The training process consists of \(J_T\) rounds and each round consists of \(J_R\) time slots. In the \(j\)-th time slot within the \(i\)-th round, the subarrays of the Tx use the beams corresponding to \(\mathcal{J}_T, i\) to transmit symbols, while the subarrays of the Rx use the beams corresponding to \(\mathcal{J}_R, j\) to receive signals. The process is terminated until the Tx completes the traversal of \(\{\mathcal{J}_T, 1, \cdots, \mathcal{J}_T, J_T\}\). To reduce the impact of noise, the cooperative training process may be repeated several times. Assuming two subarrays employed at both the Tx and the Rx, for \(\mathcal{C}_{T, 0}\) and \(\mathcal{C}_{R, 0}\) in Fig.5, an example showing the cooperative multi-subarray training process is provided in Table-I for ease of understanding.

The training overhead of our cooperative multi-subarray beam training method is \(M_T M_R/(B_T B_R)\), while the training overhead of the non-cooperative training method in [9] is \(M_T M_R\). When the training process is completed, we can obtain a training data-set \(S = \{y_1, \cdots, y_{J_T J_R}\}\). Based on the training data-set, we can construct a reduced space of BDC. The basic idea is to determine \(P\) dominant paths of the channel, where \(P\) is a small positive integer due to the sparsity of mmwave channels. The simplest strategy is the Top-\(P\) strategy, i.e., select the beam direction combinations corresponding to the \(P\) strongest elements in \(S\). More efficient strategies have been proposed in [17] to construct such spaces. Based on the reduced space, other beam training and/or beam search algorithms can be further used to perform beam training (if necessary) and search precoding matrices. The complexity analysis is provided in the next section since it also relies on the subsequent algorithms.

### V. Simulation Results

This section illustrates the performance of proposed cooperative multi-subarray beam training method. The simulation setting is given as follows. Uniform linear arrays are considered with inter-element spacing \(d = \lambda/2\). The number of paths \(L_p\) in (3) is 5. The antennas at the Tx (Rx) are equally divided into \(L_T (L_R)\) independent steerable subarrays. This set-up emulates a \(L_T \times L_R\) conventional MIMO system. The Tx sector spans 120° around boresight, while the Rx monitors a complete 180° region around boresight. The Tx (Rx) RF codebook consists of \(M_T (M_R)\) beams spread uniformly across the angular range. For comparison, besides the exhaustive beam direction search, a greedy beam direction search algorithm is also adopted [18] (See also [17]), which can further reduce the computational complexity of beam direction search. The SE performance of two beam direction search algorithms (i.e., exhaustive-search-based and greedy-search-based) based on the complete and reduced spaces is provided for comparison.

The SE performance of a \(2 \times 2\) MIMO is shown in Fig.6. We can observe that compared to the SE performance based on the complete space, the SE performance loss based on the compressed space is small for both exhaustive-search-based and greedy-search-based beam direction search algorithms. To show the superiority of proposed cooperative multi-subarray beam training method, the SE performance of a \(4 \times 4\) MIMO is shown in Fig.7. It should be pointed out that the SE curves of exhaustive-search-based beam direction search algorithm are not provided, since the complexity in this case is too high to implement. One can observe that a good SE performance can still be achieved based on the reduced space, which further shows the effectiveness/ viability of our proposals.
To reduce the training overhead and the complexity of beam direction search algorithms, we proposed a cooperative multi-subarray beam training method. From the perspective of excluding non-effective beam direction combinations, beam superposition method and criterion were proposed to construct a wide beam from the narrow beams corresponding to multiple subarrays. A cooperative beam training method was further proposed based on the criterion of beam superposition. Finally, simulation results shown the effectiveness of our proposals.

VI. CONCLUSION

To reduce the training overhead and the complexity of beam direction search algorithms, we proposed a cooperative multi-subarray beam training method. From the perspective of excluding non-effective beam direction combinations, beam superposition method and criterion were proposed to construct a wide beam from the narrow beams corresponding to multiple subarrays. A cooperative beam training method was further proposed based on the criterion of beam superposition. Finally, simulation results shown the effectiveness of our proposals.

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