Optimal Power Allocation in NOMA Systems with Imperfect Channel Estimation

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Abstract—Non-orthogonal multiple access is a promising technology for the fifth generation systems which exploits the power domain to achieve higher spectral efficiency. The performance of NOMA techniques are usually investigated under an ideal setup with perfect successive interference cancellation. However, the limitations of NOMA techniques under a setup with imperfect successive interference cancellation are not well understood. Contrary to the approaches in the literature, we examine the performance of NOMA under a non-ideal setup and propose two power allocation algorithms. The first algorithm is designed for the max-min problem whereas the second algorithm considers the heterogeneous rate requirements of users and provides solutions based on a novel rate measure. The performance of the algorithms is investigated both theoretically and numerically under a non-ideal setup with channel estimation errors. The theoretical analyses reveal that the algorithms achieve the optimum power allocation for the rate max-min problems. The numerical analyses are not only in agreement with the theoretical analyses, but also show the superiority of the proposed algorithms compared to both the conventional multiple access techniques as well as other NOMA approaches.

Index Terms—Non-orthogonal multiple access, Resource Allocation, Small Cell Networks.

I. INTRODUCTION

FUTURE fifth generation cellular networks aim to achieve a thousand-fold increase in capacity compared to the current 4G technology. To this end, various technologies such as massive MIMO (multiple-input multiple output), millimeter wave and ultra dense networks have attracted attention of many researchers [1]. Furthermore, multiple access technologies employed for future networks are expected to be more efficient in terms of spectrum and energy usage. Non-orthogonal multiple access (NOMA) is a potential multiple access technique for 5G networks which can provide high spectral efficiency. However, the performance of NOMA approaches heavily rely on power allocation and targeted data rates [2].

Orthogonal multiple access (OMA) schemes such as frequency-division multiple access (FDMA), time-division multiple access (TDMA), code-division multiple access (CDMA) and orthogonal frequency division multiple access (OFDMA) are designed based on orthogonal resource allocation in time, frequency or code domain [3]. On the other hand, NOMA schemes exploit power domain to achieve multiple access, i.e., multiple users are allowed to access the same time/frequency/code resources with different power levels. Therefore, compared to OMA techniques where resources are allocated exclusively, NOMA techniques result in a higher spectral efficiency. However, the advantage of NOMA techniques over OMA approaches is a result of successive interference cancellation (SIC). The adaptation of SIC requires receivers with increased processing capabilities. For this reason, application of NOMA techniques in the uplink are more likely to be feasible as the processing capabilities of base stations are superior compared to user terminals.

The optimal power allocation that maximizes the minimum rate for a downlink NOMA system based on the instantaneous and average channel state information, is proposed in [4]. In [5], a NOMA approach which utilizes proportional fair-based radio resource allocation is presented. An extensive comparison of NOMA with TDMA along with a power allocation scheme for NOMA with user pairing is provided in [6]. The work in [7] investigates the performance of NOMA systems with user pairing for fixed and cognitive radio based power allocation. Another NOMA approach with user pairing which allocates power dynamically based on instantaneous channel gains is proposed in [8].

The aforementioned approaches on NOMA assume an ideal communication setup in which the perfect CSI is available during SIC. Hence, the performance of NOMA is investigated when superimposed signals are perfectly separated at the receiver side by utilizing the perfect CSI. However, the availability of perfect CSI is not realistic in practical systems [9]. Furthermore, the potential of NOMA techniques can only be unveiled by comparing with OMA approaches under a non-ideal setup as OMA techniques are inherently robust to interference at the cost of reduced spectral efficiency. In other words, in the presence of imperfect SIC, there will be residual interference in the composite signal and a practical power allocation technique should be able to perform under this non-ideal communication setup. In [10], the performance of a fixed power allocation NOMA scheme is investigated under a setup with imperfect SIC. However, neither the performance limits of NOMA approach under this imperfect setup nor the design of a power allocation algorithm is considered.
In this paper, a NOMA scheme with imperfect SIC for both uplink and downlink is considered. The key contributions of this paper are as follows:

- A power allocation algorithm which can be employed in both uplink and downlink NOMA systems is proposed. The proposed algorithm is shown to provide the optimal solution to the max-min problem without relying on perfect channel state information.
- A modified version of the proposed algorithm which considers various rate requirements of users is presented. The modified algorithm is designed to provide the optimum max-min solution in terms of a new performance measure which suits the emerging applications’ ability to operate at various rates and heterogeneous rate requirements.
- The performance of the proposed algorithms is investigated both theoretically and numerically under a setup with imperfect SIC. Moreover, comparison with the existing NOMA techniques as well as TDMA reveals the superior performance of the proposed algorithms.

II. SYSTEM MODEL

We consider a NOMA system with a centralized single-antenna base station with radius $R_M$. There are $K$ uniformly distributed single-antenna users with channel gains ordered as $|h_1|^2 \leq |h_2|^2 \leq \ldots |h_K|^2$. The channel between $i$-th user and base station is modeled as $h_i = b_i / d_i^\alpha$ where $b_i$ is the Rayleigh fading channel gain, $d_i$ denotes the distance between user $i$ and base station, $\alpha$ is the path loss exponent. The channels are assumed to experience additive white Gaussian noise with zero mean and variance $\sigma_n^2$. The order of decoding is based on the order of decreasing channel gains in the uplink, i.e., users with better channel gains are decoded first, and the order is reversed in the downlink [7], [11].

A. NOMA System

The rate for user $k$ in downlink NOMA is given by [11]

$$r_k = \log_2 \left( 1 + \frac{p_k g_k}{\sum_{k=1}^{K} p_k g_k + \sigma_n^2} \right)$$

(1)

where $g_k = |h_k|^2$. The idea of NOMA is to employ power control for allocation of system resources to users. This approach allows users with inferior channel conditions to have access to more resources compared to OMA approaches.

The rate value given in (1) can only be achieved under ideal conditions. For the cases with a non-ideal setup, the achievable rate for user $i$ contains residual terms due to imperfect SIC and is given by

$$r_i = \log_2 \left( 1 + \frac{p_i g_i}{\sum_{k=1}^{i-1} \varepsilon_k p_k + \sum_{k=i+1}^{K} p_k g_i + \sigma_n^2} \right),$$

(2)

where $\varepsilon_k$ is the fraction of residual power of user $k$ after cancellation. The residual power is a result of the imperfect channel state estimations. An example comparison of capacity regions for two users achieved via TDMA and NOMA are illustrated in Fig. 1. Although the performance of NOMA under ideal setup is clearly superior to TDMA, the capacity of the system erodes rapidly with increasing residual interference.

In the uplink, the decoding order is reversed and users with better channel gains are decoded first at the receiver side. The performance of NOMA approaches depend on the SIC which requires high processing capabilities. Hence, it is a challenging task for user equipments to employ NOMA based techniques.

For the uplink, the rate of user $i$ under non-ideal setup is

$$r_i = \log_2 \left( 1 + \frac{p_i g_i}{\sum_{k=1}^{i-1} g_k p_k + \sum_{k=i+1}^{K} \varepsilon_k p_k + \sigma_n^2} \right).$$

(3)

Note that distortion due to the imperfect channel estimation acts as an interference term and is based on the transmission power.

In this work, we focus on the power allocation problem under a non-ideal setup and propose two power allocation algorithms which can be employed in both downlink and uplink. The first proposed algorithm is designed to achieve the max-min solution without relying on perfect channel state information whereas the second algorithm incorporates the heterogeneous rate requirements of users and achieves the optimal max-min solution in terms of relative error values.

III. POWER ALLOCATION

Power allocation is crucial for NOMA systems as the idea is to exploit power domain for multiple access. However, achieving optimal power allocation is a challenging task without perfect CSI information. In this section, we present two allocation algorithms designed for power control in NOMA systems. The first algorithm is tailored to achieve max-min fairness, whereas the second algorithm is an extended version which considers heterogeneous rate requirements.
A. Max-Min Power Allocation

The max-min rate problem can be expressed mathematically as determining a power allocation vector \( \mathbf{p}^* \) which is the solution of

\[
\begin{align*}
\max & \min_{k \in \mathcal{K}} r_k(\mathbf{p}) \\
\text{s.t.} & \sum_{k \in \mathcal{K}} p_k \leq p_{\text{max}}, \\
& 0 \leq p_k, \forall k \in \mathcal{K}.
\end{align*}
\]

Here \( r_k(\mathbf{p}) \) denotes the rate achieved by user \( k \) for a given power vector \( \mathbf{p} \) and \( p_{\text{max}} \) is the maximum transmission power. The problem described in (4) is known to be quasi-concave and standard optimization solvers are not applicable [4].

Let \( \mathbf{p}^* \) be the power vector which is the solution to the max-min problem, then \( r_k(\mathbf{p}^*) = r_j(\mathbf{p}^*) \) for all \((k, j) \in \mathcal{K} \times \mathcal{K}\). Hence, we want to design a power allocation algorithm with equilibrium points at \( \mathbf{p}^* \). We design our function as

\[
\mathbf{p}[t + 1] = f(\mathbf{p}[t])
\]

where \( f(\mathbf{p}[t]) = \mathbf{p}[t] \) only when \( \mathbf{p}[t] = \mathbf{p}^* \). Hence, at equilibrium we have

\[
\mathbf{F} \mathbf{r}[t] = \mathbf{r}[t]
\]

where \( \mathbf{r}[t] = [r_1[t], \ldots, r_K[t]]^T \). \( \mathbf{I} \) is the identity matrix with appropriate dimensions and \( \mathbf{F} \) is the averaging matrix with 0 diagonals and \( \frac{1}{K-1} \) at off-diagonals. The symmetric matrix \( \mathbf{I} - \mathbf{F} \) is a Laplacian matrix of an undirected graph [12]. 0 is a simple eigenvalue of \( \mathbf{I} - \mathbf{F} \) if the undirected graph is connected. (6) is satisfied when \( \mathbf{r}[t] \) is in the kernel of \( \mathbf{I} - \mathbf{F} \) which is spanned by \( \mathbf{1} \). Hence, at equilibrium points, powers are allocated such that \( r_i[t] = r_j[t] \) for all \((i, j) \in \mathcal{K}\). The power update function for user \( k \) should be

\[
p_k[t+1] = (r_k[t] - \bar{r}_k[t])a_k(\mathbf{p}[t]) + p_k[t]
\]

where \( a_k(\mathbf{p}[t]) \) is the part yet to be designed and \( \bar{r}_k[t] \) is

\[
\bar{r}_k[t] = \frac{1}{K-1} \sum_{i \neq k, i \in \mathcal{K}} r_i[t].
\]

Note that, (7) adjusts powers based on the difference between the achieved rate of user \( k \) and the average rate of remaining users. This is a desired property as it improves the rate of users with inferior channels more rapidly compared to a fixed scaling. We want to choose \( g_k(\mathbf{p}[t]) \) in a way such that this property is preserved. Furthermore, \( g_k(\mathbf{p}[t]) \) should not introduce any more equilibrium points and should rely on local measurements to make the algorithm as distributed as possible. Another desired property is that \( p_k[t+1] \) should be positive at all times. All of these properties are satisfied by choosing \( g_k(\mathbf{p}[t]) = -p_k[t]/\bar{r}_k[t] \). Then, the proposed power update rule for the max-min problem is given by

\[
p_k[t+1] = \frac{\bar{r}_k[t]}{\bar{r}_k[t]} p_k[t]
\]

The algorithm allocates the transmission powers of users with respect to the rate achieved by other users in the system. This is a crucial difference compared to other approaches in the literature where perfect CSI is usually utilized. Furthermore, computation of the achievable max-min value is not required for the algorithm, i.e., there is no requirement for a target max-min rate for the proposed algorithm. The allocation may be succeeded by a scaling process which may be required based on the application. As an example, in the cases with total power constraint \( P \), the scaled power allocation can be computed as

\[
p_k^*[t+1] = \frac{p_k[t+1]}{\sum_{i=1}^K p_i[t+1]} P.
\]

Note that, a scaling process given in (10) is required for the cases with a total power constraint which is the case for downlink transmission. However, for the uplink the power constraint is usually described as a maximum power value for each user. Both cases are considered for the theoretical analysis. The main theoretical result for the proposed update rule is presented next.

**Theorem 3.1:** The power update rule described by (9) converges to a power allocation vector satisfying, \( r_i = r_j \), for all \((i, j) \in \mathcal{K} \times \mathcal{K}\). Furthermore, the point of convergence is the optimal solution to the problem given in (4).

**Proof:** The proof for the convergence is similar to the one provided in [13]. Let \( V[t] = r_{\text{max}}[t] - r_{\text{min}}[t] \) be the difference between the maximum and minimum rate achieved by the users in the system. Without loss of generality assume that \( r_K = \arg \max_{i \in \mathcal{K}} r_i \) and \( r_1 = \arg \min_{i \in \mathcal{K}} r_i \). We need to show that the difference between the maximum and the minimum value decreases at each iteration, i.e., \( \Delta V[t] = V[t+1] - V[t] \) should be negative. \( \Delta V[t] \) is given by

\[
\begin{align*}
\Delta V[t] &= r_K[t+1] - r_1[t+1] - r_K[t] - r_1[t] \\
\log_2 &\left( 1 + \frac{\bar{r}_K[t]}{r_K[t]} g_K \right) - r_K[t] \ldots \\
\log_2 &\left( 1 + \frac{\bar{r}_1[t]}{r_1[t]} g_1 \right) + r_1[t] \\
&< \log_2 \left( 1 + \frac{\bar{r}_K[t]}{r_K[t]} g_K \right) - r_K[t] \ldots \\
&< \log_2 \left( 1 + \frac{\bar{r}_1[t]}{r_1[t]} g_1 \right) + r_1[t] \\
&= -\beta r_K[t] - \alpha r_1[t] \\
&\leq 0
\end{align*}
\]

(11a) (11b) (11c) (11d) (11e)
where

$$\beta = \frac{1}{r_K[t]} \log_2 \left( 1 + \frac{\tilde{r}_K[t]}{\tilde{r}_K[t] - 1} \frac{p_K[t] g_K}{\sum_{i=1}^{K-1} g_i p_i[t] + \sigma^2_n} \right) - 1$$

and

$$\alpha = \frac{1}{r_1[t]} \log_2 \left( 1 + \frac{\tilde{r}_1[t]}{\tilde{r}_1[t] - 1} \frac{p_1[t] g_1}{\sum_{i=2}^{K} g_i p_i[t] + \sigma^2_n} \right) - 1$$

are non-negative parameters which are only equal to zero if \(r_K[t] = r_1[t]\) which is achieved at convergence. The system can be considered as a fully connected graph where users are the nodes of the system. Combined with (11), the connected graph allows us to conclude that the algorithm converges to a solution satisfying (14) requires at least as much power component-wise [14]. Hence, using \(r_1[t] = 1\) for the problem given in (4).

**Remark 1**: The analysis provided is valid if power allocation is followed by a scaling operation. In other words, the convergence properties are preserved in the presence of a succeeding process which scales power values by a common term.

The analysis presented so far demonstrate that the proposed algorithm achieves a power allocation which provides an identical rate value \(r\) for all users. In other words, the proposed algorithm converges to a power allocation resulting in

$$r_i = r, \quad \forall i \in \mathcal{K}.$$  

(14)

In the final part of the analysis, the optimality properties of the resulting power vector are investigated.

Combining (2), (14) and using vector notation, we obtain

$$\mathbf{p} = \mathbf{R} \mathbf{H} \mathbf{p} + \eta,$$

(15)

where \(\mathbf{R} = \text{diag}(2^r - 1, \ldots, 2^r - 1)\), \(\mathbf{p} = [p_1, \ldots, p_K]^T\); \(\eta = [(2^r - 1) \sigma^2_n, \ldots, (2^r - 1) \sigma^2_n]^T\) is the normalized noise vector; and the \(K \times K\) normalized link gain matrix \(\mathbf{H} = [h_{ij}]\) is defined as

$$h_{ij} = \begin{cases} \frac{g_i}{g_j}, & j < i, \\ 0, & j = i, \\ \frac{g_i}{g_j}, & j > i. \end{cases}$$

The resulting power vector is given by

$$\mathbf{p} = (\mathbf{I} - \mathbf{R} \mathbf{H})^{-1} \eta.$$  

(16)

The solution given by (16) is Pareto efficient, i.e., any other \(\mathbf{p}\) satisfying (14) requires at least as much power component-wise [14]. Hence, using \(P = p_{\text{max}}\) in (10) results in the optimum solution, \(r^*\) for the problem given in (4).

An important feature of the algorithm is the knowledge on the value of the optimum solution \(r^*\) is not required. This is a key advantage of the proposed algorithm along with its robustness to the estimation error of the CSI values. An implementation of the proposed power update rule based on (9) is given in Algorithm 1.

An example power update and the resulting rate values using Algorithm 1 are illustrated in Fig. 2. For this particular example, the values \(\epsilon = 0.05\) and \(p_{\text{max}} = 1\)W are utilized.

**Algorithm 1** Max-min downlink power update algorithm

1: Initialize \(t \leftarrow 1, \mathbf{p} \leftarrow p_{\text{max}}/K \) // Initialization of the transmission powers and iteration number.
2: while \(t \leq \text{MAXITER}\) do
3: Users send the achieved \(r_i\) values to BS.
4: BS updates power values \(p_i[k+1] = \frac{r_k[t]}{r_k[t]} p_k[t]\), for all \(i = 1, 2, \ldots, K\).
5: The scaled power values are computed \(p_i^*[t + 1] = p_i[t+1] = \frac{p_i[t+1]}{\sum_{i=1}^{K} p_i[t+1]} p_{\text{max}}, i = 1, 2, \ldots, K\).
6: \(t \leftarrow t + 1\)
7: end while

Fig. 2. Change of rate values using Algorithm 1 for 5 users and \(p_{\text{max}} = 1\)W.

Initially, some of the users have inferior link quality and by employing the proposed algorithm the rate values converge to an identical value.

In the uplink, the total power constraint given in (4b) is different as each user equipment has a maximum transmission power constraint. Although, there are some approaches that assumes a total power constraint for uplink [8], we assume the following power constraint in the uplink

$$p_k \leq p^u_{\text{max}}, \quad \forall k \in \mathcal{K}$$  

(17)

where \(p^u_{\text{max}}\) is the maximum transmission power of the user equipments. Note that, the convergence properties of the algorithm does not depend on the power constraints. However, to achieve the optimal value the succeeding scaling process is modified as

$$p_k^*[t + 1] = \frac{p_k[t+1]}{\max_i p_i[t+1]} p^u_{\text{max}}.$$  

(18)

The scaling ensures that the maximum transmission power is equal to \(p^u_{\text{max}}\) and it is the optimal max-min solution [15].

**B. Power Allocation with Heterogeneous QoS Requirements**

The max-min solution results in a power allocation vector such that all users can achieve the same data rate. However,
users with different applications not only have different QoS requirements, but also emerging applications such as video streaming can operate at various rates. Therefore, a power allocation approach which considers heterogeneous QoS requirements of users is more suitable for the fifth generation mobile communication networks.

To model the heterogeneous QoS requirements, assume each user has a target rate \( r_i^t \). For a given \( p \), the relative rate for user \( i \) is defined as

\[
w_i(p, t) = \frac{r_i(p, t)}{r_i^t}
\]

and the relative error for user \( i \) is

\[
e_i(p, t) = 1 - w_i(p, t) = \frac{r_i^t - r_i(p, t)}{r_i^t}
\]

where \( r_i^t \) denotes the target rate value for user \( i \).

The constraints in the form of \( r_i \geq r_i^t \) are either satisfied or not. There is no notion of how close a user is to its target QoS requirement. This is especially inefficient for applications which are capable of operating at various rates. For example, a user may be able to stream video at SD quality but not HD quality. Therefore, we introduce the relative error and the relative rate as performance measures and propose a power update rule based on these measures.

The power allocation problem with heterogeneous QoS requirements based on relative errors is described by

\[
\min_{\mathbf{p}} \max_{k \in \mathcal{K}} e_k(\mathbf{p})
\]

\[
\text{s.t. } \sum_{k \in \mathcal{K}} p_k \leq p_{\max},
\]

\[
0 \leq p_k, \quad \forall k \in \mathcal{K}.
\]

For the problem given in (21), the proposed power allocation algorithm is

\[
p_k[t + 1] = \frac{\bar{w}_k[t]}{w_k[t]} p_k[t]
\]

where

\[
\bar{w}_k[t] = \frac{1}{K-1} \sum_{i \neq k, i \in \mathcal{K}} w_i[t].
\]

Algorithm 2 defines an implementation of a power allocation algorithm based on the power update (22). Note that, when users have identical target rate values, the algorithm described by Algorithm 2 reduces to (9).

An example power allocation and the resulting rate values are illustrated in Fig. 3. The change of rate values along with the desired rates of 3 users are depicted in Fig. 3a and the corresponding relative rate values are shown in Fig. 3b. The proposed algorithm achieves an identical relative rate value as users with higher relative rate values decreases their transmission powers and vice versa. For this particular example the system is feasible hence the resulting relative rate value is greater than 1.

The main result on the convergence and optimality properties of the power update algorithm defined by (22) is stated next.

Algorithm 2 QoS based downlink power update algorithm

1: Initialize \( t \leftarrow 1 \), \( p \leftarrow p_{\max} / K \) // Initialization of the transmission powers and iteration number.
2: \textbf{while } \( t \leq \text{MAXITER} \) \textbf{do}
3: \hspace{1em} Base station obtains \( e_i \) values for each user.
4: \hspace{1em} BS computes \( p_k[t + 1] = \frac{\bar{w}_k[t]}{w_k[t]} p_k[t] \), for all \( i = 1, 2, \ldots, K \).
5: \hspace{1em} The scaled power values are computed \( p_i^t[t + 1] = \frac{p_i[t+1]}{\sum_j p_j[t+1]} p_{\max}, \ i = 1, 2, \ldots, K \).
6: \hspace{1em} \( k \leftarrow k + 1 \)
7: \textbf{end while}

(a) Desired and achieved rates of users with Algorithm 2.

(b) Corresponding relative rates

Fig. 3. An example power allocation utilizing Algorithm 2 for 3 users and \( p_{\max} = 1W \).

\[\textbf{Theorem 3.2}: \text{The power update rule described by (22) converges to a power allocation vector satisfying, } e_i = e_j, \text{ for all } (i, j) \in \mathcal{K} \times \mathcal{K}. \text{ Furthermore, the point of convergence is the optimal solution to the problem given in (21).}
\]

\[\textbf{Proof}: \text{The analyses for the critical points and convergence are similar to the previous case and the result on the}
\]
convergence is stated without proof. The power update given in (22) converges to a power vector which satisfies $e_i = e_j$, for all $(i, j) \in \mathcal{K} \times \mathcal{K}$. Next, we investigate the optimality properties of the resulting power vector. First, note that

$$e_i = e_j \iff w_i = w_j, \forall i, j \in \mathcal{K} \times \mathcal{K} \quad (24)$$

which leads to

$$\log_2 \left( 1 + \frac{p_i g_i}{\sum_{k=1}^{\mathcal{K}} p_k g_k + \sum_{k=i+1}^{\mathcal{K}} \varepsilon_k p_k + \sigma^2_n} \right) = w r_i^\prime$$

\forall i \in \mathcal{K} \text{ for some } w \text{ value. In vector notation,}

$$\mathbf{p} = (\mathbf{I} - \tilde{\mathbf{R}})^{-1} \tilde{\eta}, \quad (26)$$

where $\tilde{\mathbf{R}} = \text{diag}(2^{w r_i^\prime} - 1, \ldots, 2^{w r_{\mathcal{K}}})$, $\mathbf{p} = [p_1, \ldots, p_\mathcal{K}]^T$; $\tilde{\eta} = (2^{w r_i^\prime} - 1)\sigma^2_n, \ldots, (2^{w r_{\mathcal{K}}})^\mathcal{K}$. Similar to the previous case, the resulting power vector is known to be Pareto optimal and a scaling process with $P = p_{\text{max}}$ results in the optimal solution for the problem described in (21).

The proposed algorithms are capable of operating without explicit knowledge on CSI values. The required information is on the achieved rate values or error values which are already measured at the receiving base station. Another important feature of the algorithm is that if the system is not feasible the algorithm achieves a power allocation resulting in equal relative error values.

IV. NUMERICAL RESULTS

In this section, we present the numerical results for the proposed algorithms. To verify the theoretical analyses and the results on the optimality of the proposed algorithms, Monte Carlo simulations are utilized. For comparison purposes, power allocation techniques presented in [4], [7] along with the conventional TDMA are used. The power allocation presented in [4] referred as NOMA-1 relies on the perfect CSI to find the optimal power allocation where as the approach given in [7] referred as NOMA-2 employs a fixed power allocation based on the order of channel links and number of users. The system model utilized is introduced in Section II and is similar to the one provided in [10].

The cumulative distribution probability (CDF) curves for the minimum rate achieved by different approaches are illustrated in Fig. 4. For TDMA, the results are obtained with equal time-split ratio and maximum power allocated to each user at its allocated slot. It is clear that NOMA techniques combined with SIC demonstrate better performances compared to TDMA. Among the NOMA techniques, the proposed algorithm has the best results whereas NOMA-1 provides comparable performance.

Fig. 5 depicts the average of minimum rate obtained for different $p_{\text{max}}$ values where the average is taken over 2000 setups for each $p_{\text{max}}$ value. The performance of the proposed algorithm, especially at higher transmission powers is clearly better compared to other approaches. NOMA-1 demonstrates comparable results for low power values, however as total power increases its performance rapidly erodes due to its dependence on perfect CSI information.

The performance of the algorithms with respect to number of users is illustrated in Fig. 6. Similar to the previous case, the proposed power allocation algorithm achieves the best results. The performance difference between the cases with $K = 5$ and $K = 10$ is substantially higher than the difference between $K = 10$ and $K = 20$ as a result of allocating resources among increasing number of users. Similar to the previous case all of the NOMA based techniques combined with SIC outperform TDMA.

The effect of imperfect SIC is demonstrated in Fig. 7. The example reveals the shortcomings of NOMA-1 as the performance of NOMA-1 greatly diminishes with increasing residual power resulting due to imperfect channel state information. Note that, for the case with $\varepsilon = 0$ the proposed algorithm and
Fig. 6. Average minimum rate obtained with variable number of users for $p_{max} = 8$ and $\varepsilon = 0.01$.

Fig. 7. The minimum rate achieved under a setup with imperfect SIC.

NOMA-1 achieves identical results, this is to be expected as NOMA-1 is shown to achieve the optimal solution for the perfect case. The TDMA is unaltered by the change on $\varepsilon$ values whereas NOMA-2 exhibits a slight decline. Although, the achieved average rate of the proposed algorithm decreases with increasing $\varepsilon$, the resulting performance is substantially better than other approaches.

V. Conclusions

In this paper, the max-min problem for the NOMA systems with imperfect channel state information is investigated. A power allocation algorithm for the max-min problem that achieves the optimal solution is presented. Another power allocation algorithm which considers heterogeneous rate requirements is provided as an extension to the max-min algorithm. Both algorithms are shown to achieve the optimal power allocation under a setup with imperfect SIC. The numerical analyses not only verify the theoretical results, but also demonstrate significant improvement in terms of overall system performance.

References