Auction based Distributed Resource Allocation for Delay Aware OFDM based Cloud-RAN System

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Abstract—Cloud-radio access network (C-RAN) is regarded as a promising solution to manage heterogeneity and scalability of future wireless networks. The centralized cooperative resource allocation and interference cancellation methods in C-RAN significantly reduce the interference levels to provide high data rates. However, the centralized solution will not be scalable due to the dense deployment of small cells with fractional frequency reuse by small cells, causing severe inter-tier and inter-cell interference turning the resource allocation and user association into a more challenging problem. In this paper, we propose an auction based distributed resource allocation method (ADRA) for a two-tier OFDM based C-RAN system. We investigate a joint user association, radio resource and power allocation problem for small cells underlaying a macro C-RAN system. First, we establish a queuing model in C-RAN. We then formulate an optimization problem for joint user association and resource allocation with the aim to minimize mean response time. Resource allocation, interference and queueing stability constraints are considered in the optimization problem. To solve this problem, we propose a distributed method where small cell users and small cell base stations jointly participate using the concept of auction theory. The ADRA method is evaluated via simulations by considering different ratio of bandwidth utilization.

Index Terms—Distributed Resource Allocation, User Association, Cloud-RAN, Auction Theory.

I. INTRODUCTION

The ever-growing use of smart phones and portable devices such as tablets, smart watches increases the growth of cellular Internet data traffic exponentially. According to the Cisco visual networking report, the global mobile data traffic will show 53 percent compound annual growth rate from 2015 to 2020, within 75 percent of data will be video, producing 30.6 exabytes per month by 2020 [1]. It is anticipated that within the next five years, 11.6 billion of connected devices including IoT devices, will increase network connection speeds by more than threefold, and the number of mobile-connected devices per capita will reach 1.5 by 2020 [1].

To deal with ever-increasing demand of user association and resource allocation in cellular networks, the architecture of cloud radio access networks (C-RAN) is envisioned as an attractive paradigm that takes advantage of managing large number of small cells through the centralized cloud controller, known as base band processing unit or BBU pool. Fig. 1 depicts a small cell-based C-RAN architecture where RRHs are responsible for RF signal transmission from/to users in the small cell and to/from BBU pool through fronthaul links. The access requests of users are transmitted from RRH to BBU pool for baseband processing. To satisfy the demand for large bandwidth and data rate, the optical fiber is generally considered as an ideal fronthaul link for C-RAN, whereas wired and wireless links support C-RAN backhaul [2]. The inspiring factor for such a centralized structure of C-RAN is to minimize capital expenditure (CAPEX) and operating expenditure (OPEX) cost as well as support scalability and flexibility of deployment of RRHs [3]. However, user association, cell activation, dynamic resource allocation based on users QoS requirements, workload scheduling in BBU pool, BBU-RRH mapping etc. are the major challenging issues in C-RAN.

In C-RAN, the data rate provisioning can be significantly improved due to the fractional frequency reuse performed by small cells [4], in which specific partitions of the spectrum are shared between both RRHs and MBS to alleviate the inter-tier interference. In [5], authors studied a combinatorial optimization problem for joint resource block (RB) and power allocation in OFDM based C-RAN system, in which the inter-tier interference is cancelled by imposing a constraint at RRH side. Moreover, the central cooperative interference cancellation in C-RANs can significantly reduce the interference levels to provide high data rates. However, the centralized method will not be scalable due to the dense deployment of small cells in a multi-tier system, turning the user association into a more challenging problem.

Recently, the application of auction theory to allocate resources in a distributed way has received increasing attention amongst researcher of future wireless networks [6]–[9]. A comprehensive introduction and applicability of auction theory in wireless network are provided in [6], [7]. Authors in [8] proposed a auction based framework for distributed resource allocation in a multi-tier device-to-device communication enabled network. In [9], authors proposed a combinational auction algorithm to address the user association problem in
60 GHz millimeterWave wireless access network. Different from the above works, we apply the auction theory to solve user association, radio resource and power allocation problem in an OFDM based C-RAN network. The main contributions of this paper are:

- An optimization problem for joint user association, radio resource and power allocation is formulated with the objective to minimize delay for small cell users.
- We consider user maximum power and queuing stability constraints for the joint user association and resource allocation problem.
- To solve the problem, we propose a distributed resource allocation method (ADRA) using auction theory, addressing all the aforementioned constraints. In the ADRA method, the small cell users and base stations cooperatively decide transmission alignment (e.g., radio resource and power) and service rate. Each user can select one base station based on the minimum mean response time or maximum service rate.
- Finally, the effectiveness of the ADRA method is verified through Monte Carlo simulation.

The rest of the paper is organized as follows. In Section II, the system model for user association and resource allocation in C-RAN is described. The corresponding optimization problem and analytical solution for optimal power allocation are provided in Section III. Section IV presents the auction based distributed resource allocation method. Section V provides the numerical results of our proposed method. Finally, Section VI concludes the paper.

II. SYSTEM MODEL AND ASSUMPTIONS

In this paper, we consider a OFDM based two tier C-RAN network where \( N \) number of RRHs are covered by one macro cell in an underlay manner. Each small cell user (SUE) is equipped with one antenna and each RRH has \( M \) antenna. The system supports \( K \) number of users, where SUE indexed by \( j = \{1, 2, 3, ..., K\} \) and RRH indexed by \( i = \{1, 2, 3, ..., N\} \). For simplicity, we assume that each user associates with one of the RRHs and one resource block is assigned between the user and RRH. The system supports \( R \) number of resource blocks (RB), indexed by \( r = \{1, 2, 3, ...R\} \). Fig. 2 represents the queuing network model of the two-tier C-RAN network. Each RRH has a transmission queue which receives requests from small cell users and processes the request at a pre-defined service rate. The RRH transmits the access requests of users to the BBU pool for baseband processing. The BBU pool is maintained by software defined C-RAN controller or scheduler which distributes the incoming requests to BBU servers for computation.

A. RRH Transmission Queue

Fig. 3 represents a queuing model for RRH. In the OFDM based C-RAN, RRHs receive requests in transmission time interval (TTI). At a given TTI, the RRH receives requests from \( K \) number of SUEs with Poisson arrival rate \( \lambda_1, \lambda_2, ..., \lambda_K \). The average incoming requests at \( i^{th} \) RRH is represented as \( \lambda_i = \sum_{j=1}^{K} \lambda_{i,j} \), where \( a_{i,j} \) denotes the user association, defined as,

\[
a_{i,j} = \begin{cases} 
1, & \text{if user } j \text{ is associated with RRH } i \\
0, & \text{otherwise.}
\end{cases}
\]

The arrival of scheduled requests to RRH \( i \) follows Poisson process with an average of \( \lambda_i \) and the inter-arrival service time is exponentially distributed time with rate \( \mu_i \). The service rate of each RRH is related with transmission rate that varies with time variation of channel and states of base station [10] [11]. The service process of each RRH follows an M/M/1 queuing model. The average response time of \( i^{th} \) RRH can be formulated as:

\[
T_i = \frac{1}{\mu_i - \lambda_i}. \tag{2}
\]

III. PROBLEM FORMULATION

Let \( \beta_{i,j}^r \) be the binary variable for RB allocation, updated as follows:

\[
\beta_{i,j}^r = \begin{cases} 
1, & \text{if RB } r \text{ is assigned to RRH } i \text{ on SUE } j \\
0, & \text{otherwise.}
\end{cases} \tag{3}
\]

The channel gain from RRH \( i \) to SUE \( j \) on RB \( r \) is denoted as \( h_{i,j}^r \in \mathbb{C}^{M \times 1} \), and \( W_{i,j}^r \in \mathbb{C}^{M \times 1} \) denotes the precoding vector for SUE \( j \) to RRH \( i \) on RB \( r \). The allocation power from RRH \( i \) to user \( j \) on RB \( r \) is denoted as \( P_{i,j}^r \in (0, P_{i,j}^{max}) \), where \( P_{i,j}^{max} \) is the maximum power of RRH \( j \). The SINR achieved by SUE \( j \), attached to RRH \( i \) on RB \( r \) can be written as:

\[
\gamma_{i,j}^r = \frac{h_{i,j}^r P_{i,j}^r}{\sum_{u \neq i, v \neq j} h_{u,v}^r P_{u,v}^r + \eta_0}, \tag{4}
\]

where \( \eta_0 \) represents the zero mean and unit variance additive white Gaussian noise (AWGN) power. According to the Shanon’s formula, the service rate for each user that is
associated to \( i \)th RRH can be obtained as:

\[
\mu_{i,j} = \Delta B \beta_{i,j}^r \log_2(1 + \gamma_{i,j}^r)
\]

where \( \Delta B \) represents the available bandwidth of each RB. The service time of RRH \( i \) can be represented by

\[
\overline{\tau}_i = \sum_{j=1}^{\kappa} a_{i,j} \mu_{i,j}
\]

According to (2) the average response time of \( i \)th RRH can be formulated as:

\[
\overline{T}_i = \frac{1}{\mu_i - \overline{\tau}_i} = \frac{1}{\sum_{j=1}^{\kappa} a_{i,j} \mu_{i,j} - \sum_{j=1}^{\kappa} a_{i,j}}.
\]

Our objective is to minimize the response time of each RRH by optimizing joint user, resource block and power allocation. The optimization problem can be formulated as:

\[
\text{(P1)} \quad \min_{\Gamma_{r,j}^r, \mu, \beta_r, P_i^r} \overline{T}_i
\]

subject to:

C1: \[ \sum_{i=1}^{N} a_{i,j} = 1, \quad \forall j \in \mathcal{K}, \]

C2: \[ \sum_{r=1}^{R} a_{i,j} \beta_{i,j}^r \leq 1, \quad \forall i, j, \]

C3: \[ a_{i,j} \beta_{i,j}^r \gamma_{i,j}^r > \gamma^h, \quad \forall i, j, \]

C4: \[ a_{i,j} \beta_{i,j}^r \gamma_{i,j}^r \leq 1, \quad \forall \gamma \in \mathcal{R}, \forall (u, v) \in \mathcal{S}_i, \]

C5: \[ \sum_{j=1}^{\kappa} \sum_{r=1}^{R} a_{i,j} \beta_{i,j}^r P_{i,j}^r \leq \bar{P}_{i,j} \leq P_{i,j}^\max, \quad \forall i \in \mathcal{N}, \]

C6: \[ \bar{\mu}_i - \overline{\tau}_i, \quad \forall i \in \mathcal{N}, \]

C7: \[ \beta_{i,j}^r \in \{0, 1\}, \quad \forall j \in \mathcal{K}, \forall \gamma \in \mathcal{R}, \]

where the constraint C1 refers that each user can associate with one RRH. Constraint C2 refers to resource block allocation constraint. For simplicity, we assume that each user and RRH connection utilizes one RB for data transmission. Constraint C3 refers that each RRH assigns a RB to serve a user only when its instantaneous SINR exceeds a threshold value (e.g., \( \gamma^h \)). Constraint C4 mitigates co-tier interference. \( S_i \) represents the set of users that are associated with RRH \( i \). According to the constraint C4, users in \( S_i \) utilize different RB for data transmission. The total power constraint of each RRH is represented by constraint C5. The constraint C6 maintains the queueing stability at each RRH, i.e., the arrival rate should not be greater than the service rate. The constraint C7 represents that the decision variable for user association and RB allocation are the binary variables. Finally, C8 defines the non-negativity condition of transmit power.

A. Relaxation to Fractional Resource Allocation

It is worth noting that the constraints C6 and C7 turn the problem P1 into a mixed integer non-linear program (MINLP) with the non-convex feasibility set. The optimization problem P1 is computationally intractable and is a NP-hard problem. We relax the problem P1 by replacing non-convex constraint with a convex constraint. First, we relax the constraint C7 with the convex constraint by assuming that time sharing approach [12] of RB allocation \( 0 \leq \beta_{i,j}^r \leq 1 \). We introduce two new variables \( \Gamma_{r,j}^r = a_{i,j} \beta_{i,j}^r \in (0, 1] \) and \( A_{i,j}^r = \Gamma_{r,j}^r \gamma_{i,j}^r \in (0, 1] \). \( \Gamma_{r,j}^r \) represents both user association and sharing factor of resource block and denotes the portion of time the RB \( r \) is allocated to an user \( j \) and RRH \( i \). Next, we relax the constraint C6 by assuming no interference, \( \gamma = \frac{\beta_{i,j}^r P_{i,j}^r}{\eta_0} = \frac{\beta_{i,j}^r P_{i,j}^r}{\Gamma_{r,j}^r} + \frac{\beta_{i,j}^r P_{i,j}^r}{\gamma_{i,j}^r} \) and the service rate of each RRH becomes \( \overline{\tau}_i = \Delta B \sum_{j=1}^{\kappa} \Gamma_{r,j}^r \log_2 \left( 1 + \frac{\beta_{i,j}^r P_{i,j}^r}{\Gamma_{r,j}^r} \right) \). The primal formulation in P1 can be expressed in an equivalent form by including new a set of variables \( \Gamma_{r,j}^r \) and \( A_{i,j}^r \). The relaxed problem can be represented by:

\[
\text{(P2)} \quad \min_{\Gamma_{r,j}^r, A_{i,j}^r, \mu, \beta_r, P_i^r} \overline{T}_i = \frac{1}{\mu_i - \overline{\tau}_i}
\]

subject to:

C1: \[ \sum_{i=1}^{N} \Gamma_{r,j}^r = 1, \quad \forall j \in \mathcal{K}, \]

C2: \[ \sum_{r=1}^{R} \Gamma_{r,j}^r \leq 1, \quad \forall i, j, \]

C3: \[ \Gamma_{r,j}^r \delta_{i,j}^r > \gamma^h, \quad \forall i, j, \]

C4: \[ \Gamma_{r,j}^r + \Gamma_{r,j}^r \leq 1, \quad \forall r \in \mathcal{R}, \forall (u, v) \in \mathcal{S}_i, \]

C5: \[ \sum_{j=1}^{\kappa} \sum_{r=1}^{R} A_{i,j}^r \leq P_{i,j}^\max, \quad \forall \gamma \in \mathcal{N}, \]

C6: \[ \overline{\tau}_i \leq \mu_i, \quad \forall i \in \mathcal{N}, \]

C7: \[ \beta_{i,j}^r \in (0, 1], \quad \forall i \in \mathcal{K}, \forall \gamma \in \mathcal{R}, \]

As the number of resource block becomes relatively large, the duality gap of any optimization problem satisfying time sharing condition to be negligible. The solution of relaxed optimization problem P2 is asymptotically optimal because it satisfies the time sharing condition [12]. The relaxed optimization problem P2 is convex; the objective function is concave and all the constraints are affine. Therefore, interior point methods can solve this problem [13].

\[
\text{(P3)} \quad \max_{\Gamma_{r,j}^r, A_{i,j}^r} \overline{T}_i = \frac{1}{\mu_i - \overline{\tau}_i}
\]

subject to:

C1 to C2

C3: \[ -\Gamma_{r,j}^r \delta_{i,j}^r + \gamma^h \leq 0, \quad \forall i, j, \]

C4 to C7

To observe the nature of resource block and power allocation, we formulate an equivalent problem P3 and use Karush-Kuhn-Tucker (KKT) optimality and define the following Lagrangian function:
\[ L(\Gamma, \mathcal{A}, \sigma, \varsigma, v, \phi, \varphi, \psi) = \Delta B \sum_{j=1}^{K} \Gamma_{i,j}^{r} \log_2 \left( 1 + \frac{\delta_{i,j}^r A_{i,j}^r}{\Gamma_{i,j}^{r}} \right) - \sum_{j=1}^{K} \lambda_j \sum_{j=1}^{K} \frac{\Gamma_{i,j}^{r}}{\beta_{i,j}^{r}} \]
\[ + \sum_{j=1}^{K} \sigma_j \left( 1 - \sum_{i=1}^{N} \frac{\Gamma_{i,j}^{r}}{\beta_{i,j}^{r}} \right) + \sum_{j=1}^{K} \sum_{i=1}^{N} \xi_{i,j} \left( 1 - \sum_{r=1}^{R} \Gamma_{i,j}^{r} \right) \]
\[ + \sum_{r=1}^{R} u_r (0 + \Gamma_{i,j}^{r} \delta_{i,j}^{th} - \gamma^{th}) + \sum_{i=1}^{N} \sum_{r=1}^{R} \phi_i r (1 - \Gamma_{u,i}^{r} - \Gamma_{v,i}^{r}) \]
\[ + \sum_{i=1}^{N} \varphi_i \left( \Delta B \sum_{j=1}^{K} \Gamma_{i,j}^{r} \log_2 \left( 1 + \frac{\delta_{i,j}^r A_{i,j}^r}{\Gamma_{i,j}^{r}} \right) - \sum_{j=1}^{K} \lambda_j \sum_{j=1}^{K} \frac{\Gamma_{i,j}^{r}}{\beta_{i,j}^{r}} \right), \]
\[ (11) \]

where \( \varphi \) and \( \psi \) are the vector of Lagrange multipliers associated with power and queueing stability requirements for cellular and SUEs respectively. Similarly, \( \sigma, \varsigma, v, \phi \) are the Lagrange multipliers for the constraints C1-C4. Differentiating (11) with respect to \( A_{i,j}^r \), we obtain the following power allocation of SUE \( i \) over RB \( r \):
\[ P_{i,j}^{r} = \frac{A_{i,j}^{r}}{\Gamma_{i,j}^{r}} = \left[ \frac{1}{\delta_{i,j}^r} \right]^+, \]
\[ (12) \]

where \( \xi = \frac{\Delta B(1+\phi)}{\ln \sigma} \) and \( [\varepsilon]^+ = \max(\varepsilon, 0) \), which is a multi-level water filling allocation [12].

Although we found a close form solution to optimal power allocation using Lagrange multiplier, it is still difficult to solve for optimal radio resource allocation from (11) due to the mathematical intractability. In the next section, we present a auction based distributed solution that satisfies all the constraints of the original problem P1 under the assumption that the system is feasible, i.e., given the network size, number of RB and SINR threshold value.

IV. A UCTION B ASED D ISTRIBUTED R ESOURCE A LLOCATION (ADRA)

In the auction based resource allocation procedure, we assume that small cell users (e.g. SUEs), small cell base stations (e.g. SBS/RRH) are the agents. An auctioneer is a software defined module, which resides in either MBS or BBU pool for controlling resources in C-RAN. It is assumed that all SUEs and RRHs within the C-RAN are always connected to the auctioneer using the control plane. The exchange of bidding information and the detail auction procedure are illustrated in Fig. 4 and works as follows:

- In the first step, whenever a user receives the pilot signal from base stations, it generates a bid information and sends this information to base stations through the auctioneer. The bid information contains two types of information, i.e., F1 and F2. F1 represents the data rate requirement based on which applications are running in the user side. The data rate requirement (in bps) serves as an arrival rate to the base stations. F2 contains the information about the candidate base station list. A base station becomes a candidate for users, when the following condition is satisfied:

\[ F_r (r < R) = \frac{\pi r^2}{\pi R^2} <= 1, \]

where \( r \) represents the distance between SUE and RRH and \( R \) denotes the radius of RRH.

- In step two, the base station executes Algorithm 1 to determine transmission alignment (i.e., transmission power and resource block), service rate for each user. Algorithm 1 satisfies all the constraints of problem P1. If the base station finds a feasible assignment for users, it sends a bid information to users which contains the expected service rate and mean response time of the base station.

Algorithm 1: Auction-based resource allocation

**Input:** receives bid information F1 and F2, containing the information about the initial user association (a), arrival rate (\( \lambda \)) for all users and available RBs and corresponding threshold value (\( \gamma^{th} \)).

**Output:** return the bid information F3 which represents the mean service rate (\( \bar{\mu} \)) and mean response time of base station (\( \bar{T} \)).

**I. Initialization:** From initial user association matrix \( a \), The \( j^{th} \) RRH estimates mean arrival rate \( \bar{\mu}_j \) and sets equal power level for users \( P_{i,j} = \frac{\pi r^2}{\pi R^2} \).

\[ \bar{\mu}_i \leftarrow \inf \left\{ \mu_i : \mu_i > 0 \right\} \]

**II. Iteration:**

for \( j \leftarrow 1 \) to \( K \) do

if \( \bar{\mu}_j > \bar{\mu}_0 \) then // Check constraint C6

if \( a_{i,j} = 1 \) then

for \( r \leftarrow 1 \) to \( R \) do

if \( P_{i,j}^r = 0 \) then // Check constraint C2 and C4

Estimate \( \gamma_{i,j}^r \) using equation (4)

if \( \gamma_{i,j}^r = 0 \) then // Check constraint C3

\[ \gamma_{i,j}^r \leftarrow 1 \]

Estimate service rate of each user (\( \mu_{i,j} \)) and mean service rate of RRH (\( \bar{\mu} \)) using equation (5) and (6).

Calculate response time of RRH (\( \bar{T} \)) using equation (7).

break;

else

\[ \mu_{i,j} \leftarrow 0 \]

end

end

end

return \( \bar{\mu} \) and \( \bar{T} \).

- In the third step, each user can choose one base station based on the maximum offered service rate or minimum mean response time information. The user acknowledges the selected base station by F4 message.

- Upon receiving the F4 message, the base station updates the transmission alignment and sends the allocated RB list and corresponding new SINR threshold value to the MBS.

V. SIMULATION RESULTS

In this section, the performance of ADRA is investigated and evaluated. In the simulation model, as shown in Fig. 5, we consider 120m \( \times \) 100m area, where one macro base station is underlaid by 6 to 10 small cell base stations. The locations
utilization is system response time becomes the lowest when the bandwidth
to refer the frequency re-used ratio of C-RAN system. The
P1 and C4 of problem
C-RAN system, we consider that SUE utilizes the RBs of
bandwidth utilization, as shown in Fig. 6. In the two-tier
method with mean arrival rate and different percentage of
C-RAN system.
In this paper, we proposed an auction based distributed user
association and resource allocation method for two-tier OFDM
based delay aware C-RAN system. The proposed ADRA
method satisfied the user association, resource allocation and
maximum power constraints as well as the queuing stability
constraint. In addition, the ADRA method associates one
user with one base station based on the minimum mean
response time or maximum offered service rate through the
auction procedure. The results showed that the performance of
proposed ADRA scheme becomes the best when the system
support 100 percent bandwidth utilization.

Fig. 4: Auction based distributed resource allocation method.

TABLE I: Simulation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total no. of small cells</td>
<td>6 – 10</td>
</tr>
<tr>
<td>Total no. SUEs</td>
<td>20 – 30</td>
</tr>
<tr>
<td>Total no. of RB</td>
<td>50</td>
</tr>
<tr>
<td>RB bandwidth</td>
<td>180 kHz</td>
</tr>
<tr>
<td>System bandwidth</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Radius of small cell</td>
<td>10 m</td>
</tr>
<tr>
<td>Minimum data rate</td>
<td>50-120 kbps</td>
</tr>
<tr>
<td>requirements</td>
<td></td>
</tr>
<tr>
<td>Number of MUEs</td>
<td>10 – 20</td>
</tr>
<tr>
<td>Transmission power of RRH</td>
<td>30 dBm</td>
</tr>
<tr>
<td>Path-loss exponent</td>
<td>4</td>
</tr>
<tr>
<td>Noise power spectrum</td>
<td>−144 dBm/Hz</td>
</tr>
</tbody>
</table>

of RRHs, SUEs and MUEs are modeled using spatial Poisson point process (PPP) with predefined intensity values. The settings for the simulation parameters are shown in Table I. The simulation is averaged over 100 iterations. The minimum data rate requirement is considered as the arrival rate of a user. The performance of our proposed method is evaluated in terms of mean response time and service rate of the entire C-RAN system.

VI. CONCLUSION

In this paper, we proposed an auction based distributed user association and resource allocation method for two-tier OFDM based delay aware C-RAN system. The proposed ADRA method satisfied the user association, resource allocation and maximum power constraints as well as the queuing stability constraint. In addition, the ADRA method associates one user with one base station based on the minimum mean response time or maximum offered service rate through the auction procedure. The results showed that the performance of proposed ADRA scheme becomes the best when the system support 100 percent bandwidth utilization.
Considering only the power allocation (e.g., $A_{i,j}^r$) part from (14):

$$L(A_{i,j}^r) = \Delta B \sum_{j=1}^{\mathcal{K}} \Gamma_{i,j}^r \left(1 + \sum_{i=1}^{N} \psi_i \right) \log_2 \left(1 + \frac{\delta_{i,j}^r A_{i,j}^r}{\Gamma_{i,j}^r} \right)$$

$$- \sum_{i=1}^{N} \varphi_i \sum_{j=1}^{\mathcal{K}} \sum_{r=1}^{\mathcal{R}} A_{i,j}^r$$

To maximize P3 for any given $\Gamma_{i,j}^r$ is equivalent to differentiate $L(A_{i,j}^r)$ with respect to $A_{i,j}^r$ and set the result to zero. That is:

$$\frac{\partial L}{\partial A_{i,j}^r} = 0$$

$$\frac{\partial B}{\partial \Gamma_{i,j}^r} \left(1 + \psi_i \right) \delta_{i,j}^r \varphi_i = 0$$

$$P_{i,j}^* = \frac{A_{i,j}^r}{\Gamma_{i,j}^r} = \left[ \frac{\Delta B \left(1 + \psi_i \right)}{\ln \varphi_i} - \frac{1}{\delta_{i,j}^r} \right]^+$$

REFERENCES


