Study on Set Partitioning 64APSK Coded Modulation Design Method Based on Channel Capacity

Yuki Koizumi, Yoichi Suzuki, Masaaki Kojima, Kyoichi Saito, Shoji Tanaka
Science and Technology Research Laboratories
NHK (Japan Broadcasting Corporation)
Tokyo, Japan
Email: koizumi.y-hq@nhk.or.jp

Abstract—We are researching set partitioning 64APSK coded modulation for satellite transmission. We previously developed a constellation-design method based on channel capacity [6] and evaluated the transmission performance under the condition that the constellation was restricted to circumferential arrangement [4]. We confirmed that this conventional method exhibited better transmission performance than DVB-S2X [7]. In this paper, we expanded the numbers of circles of 64APSK from four to seven. Especially, applying the number of five shows good channel capacity. We also applied the design scheme whose constellation points-assignment is not restricted to circumferential arrangement because of the free constellation points-assignment in the IQ plane. We show that the proposed 64APSK applying these design schemes achieved better transmission performance than conventional method through computer simulation under the AWGN model.

Keywords—64APSK; Coded modulation; Set partitioning; Channel capacity; LDPC code; Satellite transmission

I. INTRODUCTION

NHK started test satellite broadcasting of 4K/8K ultra-high definition television (UHDTV) in August 2016. In test broadcasting, ISDB-S3 [1] is adopted as the transmission system and can broadcast one 8K program or three 4K programs with the same bandwidth (34.5 MHz) as current satellite broadcasting. By using the symbol rate of 33.7561 M baud, roll-off factor of 0.03, 16APSK modulation scheme, and LDPC coding rate of 7/9 as the transmission parameters, the required transmission capacity of 100 Mbps for test broadcasting is possible. Currently, we are developing equipment for practical broadcasting of 4K/8K UHDTV in 2018.

NHK is also conducting research on new broadcasting services, e.g., a full-spec 8K system corresponding to a frame frequency of 120 Hz and viewable three-dimensional television that does not require special glasses. To develop such services, a further increase in the transmission capacity is required. We are conducting research to expand the transmission capacity for satellite transmission. We are investigating multi-level coded modulation as an approach to expand this capacity and have confirmed that the transmission performance for satellite transmission. We are investigating multi-level coded modulation as an approach to expand this capacity and have confirmed that the transmission performance for satellite transmission. We are investigating multi-level coded modulation as an approach to expand this capacity and have confirmed that the transmission performance for satellite transmission. We are investigating multi-level coded modulation as an approach to expand this capacity and have confirmed that the transmission performance for satellite transmission.
probability-density function determined from the C/N and the minimum Euclidian distance, and $\sigma^2$ is the noise power under the AWGN model.

By carrying out constellation design and bit allocation to maximize $T$, the minimum Euclidian distance can be increased and transmission performance improved.

$$T = \log_2 M - \frac{1}{M} \sum_{i=0}^{M-1} p(y|x_i) \log_2 \left[ \frac{1}{M} \sum_{k=0}^{M-1} p(y|x_k) \right]$$  \hspace{1cm} (1)

$$p(y|x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{1}{2\sigma^2} \right)$$  \hspace{1cm} (2)

$$x_i = I_i + jQ_i$$  \hspace{1cm} (3)

III. CONSTELLATION DESIGN

A. Outline

Since a general constellation in APSK has a circumferential arrangement, it cannot be uniquely determined, unlike QAM with a square lattice constellation. There are many possible combinations of constellations depending on how the constellation points are arranged on the IQ plane. In the 64APSK constellation design, it is required to determine design parameters so that $T$ is maximized.

The number of these combinations can be expressed using (4), where $S$ is the number of circles (the relation $M \geq S$ holds), and $h$ to $k$ are defined as the index of sigma calculation for $S = 4–7$.

$$\sum_{k=0}^{M-S-1} \sum_{j=0}^{M-S-1} \sum_{i=0}^{M-S-1} \sum_{h=0}^{M-S-1} (M-S+1-k-j-h)(h+1)$$  \hspace{1cm} (4)

Table I shows the number of possible constellation patterns for each number of circles. We do not consider design condition 2) and show the results of simple counting such that one or more constellation points are arranged on each circle. The number of these combinations can be expressed using (4), where $S$ is the number of circles (the relation $M \geq S$ holds), and $h$ to $k$ are defined as the index of sigma calculation for $S = 4–7$.

Table I. NUMBER OF POSSIBLE CONSTELLATION PATTERNS

<table>
<thead>
<tr>
<th>$S$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patterns</td>
<td>39,711</td>
<td>595,665</td>
<td>7,028,847</td>
<td>67,945,521</td>
</tr>
</tbody>
</table>

Fig. 1. Design outline

B. Circular Constellation

Under the design conditions discussed below, design parameters a) to d) are determined, and a 64APSK constellation is designed.

<Design conditions>

1) Constellation points are arranged equidistant on a concentric circle
2) Number of constellation points arranged on each circle is even
3) Standardize the average power to 1
4) Designed C/N is 16 dB

In a general 64APSK constellation design, the constellation is limited to circumferential arrangement. However, to achieve efficient transmission with high $T$, it is not necessary to set such a constraint condition. In Section III C, we discuss the constellation-design results when there is no limit to circumferential arrangement. The constellation is designed by solving the optimization problem that maximizes $T$ described below ((5) to (8)).

A. Design conditions

1. Number of circles allocating constellation points (4-7)
2. Number of constellation points arranged on each circle
3. Constellation-point phase ($\theta_0-\theta_7$)
4. Radius ratio between circles ($\gamma_1-\gamma_6$)

The radii of 1st-7th circles are $r_1-r_7$

Radius ratio is defined as $\gamma_1=r_2/r_1$, $\gamma_2=r_3/r_1$, …, $\gamma_6=r_7/r_1$ with $r_1$ as the standard
For S=4, the number of patterns can be obtained by calculating the first sigma in (4). Similarly, for S = 5–7, double, triple, and quadruple sigma are calculated, respectively. The number of constellation patterns shown in Table I is too large, so we need to decrease it. Thus in the process of calculating T, we found a borderline that cannot be expected to expand, even if the number of constellation patterns is further increased. In this way, we narrow down the number of constellation patterns to that shown in Table II. The number of candidate constellation points arranged on each circle is also shown. We select the number of constellation points to arrange on each circle from Table II so that 64 constellation points can be arranged on the IQ plane.

### Table II. Number of Constellation Points Arranged on Each Circle

<table>
<thead>
<tr>
<th>S</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st circle</td>
<td>4,6,...,12,14</td>
<td>4,6,...,12,14</td>
<td>4,6,8,10,12</td>
<td>4,6,8,10</td>
</tr>
<tr>
<td>2nd circle</td>
<td>4,6,...,18,20</td>
<td>4,6,...,18,20</td>
<td>4,8,12,16,20</td>
<td>4,8,12,16,20</td>
</tr>
<tr>
<td>3rd circle</td>
<td>2,4,...,52,54</td>
<td>2,4,...,46,48</td>
<td>4,8,12,16,20</td>
<td>4,8,12,16,20</td>
</tr>
<tr>
<td>4th circle</td>
<td>2,4,...,52,54</td>
<td>4,6,...,48,50</td>
<td>4,8,12,16,20</td>
<td>4,8,12,16,20</td>
</tr>
<tr>
<td>5th circle</td>
<td>-</td>
<td>4,6,...,48,50</td>
<td>4,8,...,40,44</td>
<td>4,8,12,16,20</td>
</tr>
<tr>
<td>6th circle</td>
<td>-</td>
<td>-</td>
<td>2,4,...,42,44</td>
<td>4,8,...,36,40</td>
</tr>
<tr>
<td>7th circle</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2,4,...,38,40</td>
</tr>
<tr>
<td>Number of constellation patterns</td>
<td>1107</td>
<td>9000</td>
<td>2652</td>
<td>5122</td>
</tr>
</tbody>
</table>

The constellation for each S is designed according to Table II. First, the Ts of all constellation patterns are calculated, and the constellations with the maximum T for S = 4–7 are determined. Next, the phases and radius ratios of the constellations obtained for each S are optimized, and the optimal constellations are determined. Table III lists the constellation-design results, Fig. 2 shows the radius ratios for each S, Fig. 3 shows the T per S, and Figs. 4 (a) to (d) indicate the designed constellations for each S. From these results, we confirm that the T of 64APSK with five circles is maximized.

As the S is increased, the number of constellation points arranged per circle can be reduced. It is possible to expand the minimum Euclidean distance between constellation points arranged on each circle. In contrast, the radius ratios of each circle decrease because of standardizing the average power to 1. The increase in amplitude with increase in S is suppressed by decreasing the radius ratios. Fig. 2 shows that the radius ratios decrease and the circumferential intervals become dense with six or more circles. Therefore, to expand T, it is necessary to design a constellation so that the minimum Euclidean distance of constellation points on a circle and between circles increase together. The constellation with five circles meets this condition; thus, it has maximum T.

### Table III. Constellation-Design Results

<table>
<thead>
<tr>
<th>S</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constellation</td>
<td>(12, 16, 18, 18)</td>
<td>(6, 12, 14, 16, 16)</td>
<td>(6, 12, 12, 16, 16, 2)</td>
<td>(6, 8, 12, 16, 4, 16, 2)</td>
</tr>
<tr>
<td>Phase (θ₁−θ₇)[deg.]</td>
<td>(15, 22, 25, 20, 10)</td>
<td>(45, 30, 30, 76, 22, 55, 11, 35)</td>
<td>(45, 30, 15, 22, 25, 11, 25, 90)</td>
<td>(45, 28, 5, 29, 9, 11, 35, 45, 11, 25, 90)</td>
</tr>
<tr>
<td>Radius ratio (γ₁−γ₆)</td>
<td>(2.00, 2.93, 4.05)</td>
<td>(2.27, 3.37, 4.49, 6.01)</td>
<td>(2.20, 3.22, 4.24, 5.67, 6.79)</td>
<td>(2.03, 2.92, 4.28, 5.09, 6.12, 7.33)</td>
</tr>
<tr>
<td>T [bps/Hz]</td>
<td>5.0839</td>
<td>5.0927</td>
<td>5.0770</td>
<td>5.0486</td>
</tr>
</tbody>
</table>
We discuss designing a constellation without being restricted to circumferential arrangement so that $T$ becomes maximum at $C/N = 16$ dB, which can be expressed as the optimization problem to maximize $T$ as follows (5) to (8). It is possible to design a constellation that is not restricted to circumferential arrangement by finding the optimal solution using an iterative method, which converges from arbitrarily set initial values to the optimal solution by sequentially updating a solution to satisfy the objective function. The initial values of $x_i$ are set to the constellation discussed in Section III B with five circles. The integration interval of output signal $y$ was narrowed from the infinite IQ plane ($-\infty \leq y \leq \infty$) to $-2 \leq y \leq 2$ considering computing capacity. Under the design condition of the average power of 1, we verified that even if the integration interval is narrowed there is no effect on calculation accuracy. Fig. 5 shows the 64APSK constellation obtained from the optimization problem. To illustrate that this constellation differs from that shown in Fig. 4 (b), the concentric circles in Fig. 4 (b) are also shown in Fig. 5. Although the constellation shown in Fig. 5 resembles a circumferential arrangement, each constellation point, whose IQ coordinates are given in the Appendix, is randomly arranged on the IQ plane so that the minimum Euclidean distance expands. The $T$ of this constellation and those of the constellation with five circles and four circles (Section III B) are shown in Table IV. By designing a constellation without being restricted to circumferential arrangement, we can achieve a $T$ of 5.1055 bps/Hz, which exceeds those of the constellations with the circumferential arrangement.

$$
\text{Maximize } T = \log_2 M - \frac{1}{M} \sum_{j=0}^{M-1} \int p(y | x_j) \log_2 \left( \frac{\sum_{k=0}^{M-1} p(y | x_k)}{\sum_{k=0}^{M-1} p(y | x_k)} \right) dy \tag{5}
$$

$$
\text{Subject to } M = 64
\tag{6}
\quad -2 \leq y \leq 2
\tag{7}
\quad \frac{1}{M} \sum_{j=0}^{M-1} x_j^2 = 1
\tag{8}
$$

### Table IV. Channel Capacity of Each Constellation

<table>
<thead>
<tr>
<th>Constellation-design method</th>
<th>Constellation not limited to circumferential arrangement (Section III C)</th>
<th>Circular constellation (Section III B)</th>
<th>Conventional method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel capacity (T) [bps/Hz]</td>
<td>5.1055</td>
<td>5.0927</td>
<td>5.0839</td>
</tr>
</tbody>
</table>
V. DESIGN OF LDPC CODE

We discuss the design of the LDPC codes to be applied to each bit such that the overall coding rate becomes 4/5 and the steep decoding result is obtained at C/N = 16 dB. The LDPC inner code applies the same parity check matrix structure as ISDB-S3. The BCH outer code for each bit is set to the BCH (65535, 65167, t=23) code. The number of LDPC decoding iterations is set to 50. As shown in Fig. 7, since the BER of the 6th bit is 2.54×10⁻⁷ at C/N = 16 dB, error correction is possible only with the BCH code. For the 1st to 5th bits, we determine each LDPC coding rate so that the best decoding result from the simulation model described below (Fig. 9) is obtained at C/N = 16 dB. Fig. 8 shows an example of the LDPC coding rate for each bit. The frame structure conforms to ISDB-S3 [1], and the code length is 44880 bits.

VI. COMPUTER SIMULATION RESULTS

A. Simulation model

We calculate the C/N vs. BER characteristics of the designed 64APSK coded modulation under the AWGN model through computer simulation. We used a 25th-order pseudo-random binary sequence (PRBS) as the transmission bit stream in the BER calculation. Fig. 9 shows the simulation model.

B. C/N vs. BER characteristics

Fig. 10 shows the C/N vs. BER characteristics of the 64APSK coded modulation by applying the error correction code indicated in Fig. 8. The BERs are calculated in the range of 10⁻⁸ to 10⁻¹⁰, and the points of BER = 1×10⁻¹¹ are plotted by linear interpolations from the results of BER calculations. The C/N at BER = 1×10⁻¹¹ is defined as the required C/N. As shown in Fig. 10, the required C/N of the proposed method is 15.6 dB, which makes it possible to improve transmission performance by about 0.1 and 0.4 dB compared to the conventional method and DVB-S2X’s 64APSK by applying the same LDPC code, respectively.

VII. CONCLUSION

By designing a 64APSK constellation that is not restricted to circumferential arrangement, we achieved a T of 5.1055 bps/Hz, which is higher than those of the constellations with the circumferential arrangement. As a result, the proposed 64APSK coded modulation with the designed constellation can achieve better transmission performance than the conventional method and DVB-S2X’s 64APSK.
Table V lists the IQ coordinates of the 64APSK constellation obtained from the optimization problem discussed in Section III C.