Robust widely linear beamforming via a shrinkage method for signal steering vector estimation

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Abstract—The robust adaptive beamforming (RAB) problem for noncircular signals with the desired signal’s steering vector (SV) mismatch is considered. As we know, noncircular signals are widely used in the satellite communication and radio communication. Some existing approaches for estimating the extended SV of the desired signal are based on the second-order cone programming (SOCP), which results in a high computational cost. In this paper, we propose a novel robust widely linear (WL) beamforming algorithm by using a low-complexity shrinkage-based approach. The augmented interference-plus-noise covariance matrix (IPNCM) is reconstructed first by using the SVs corresponding to the interference region. Then, a modified oracle approximating shrinkage (OAS) method is applied to estimate the desired signal’s extended SV. Only the prior knowledge of the antenna array geometry and the angular sector in which the desired signal is located are utilized in the proposed method. Numerical simulations show that the proposed algorithm outperforms the existing robust WL beamforming methods.

I. INTRODUCTION

Adaptive beamforming techniques can effectively suppress interferences and noises. It has been widely used in wireless communications, sonar, radar, et al. [1, 2]. Meanwhile, the conventional adaptive beamformer is also well-known to be sensitive to the desired signal’s steering vector (SV) mismatch and will suffer severe performance degradation, especially when the training data contains the desired signal. Therefore, many robust adaptive beamformers have been developed in the past decades [1, 3]. We know that the robust beamformers in [3] are based on the covariance matrix (CM) \( \mathbf{R}_x \) of the observations. According to [4], the second-order (SO) statistics circular complex observations are uniquely determined by \( \mathbf{R}_x \), while the SO statistics of noncircular complex observations are additionally determined by the conjugate CM \( \mathbf{C}_x \). Thus, for noncircular observations, it makes more sense to take noncircularity into account to design more suitable beamformers.

Noncircular signals are usually encountered in the context of satellite communication and radio communication, such as M-ary amplitude shift keying (MASK), amplitude modulated (AM), binary phase-shift keying (BPSK) and unbalanced quaternary phase shift keying (UQPSK) [5–8]. In order to exploit the SO noncircularity of complex signals, many widely linear (WL) adaptive beamformers have been proposed [8–15]. Chevalier et al. have proposed the optimal WL minimum variance distortionless response (MVDR) in [13] and analyzed its performance in [16]. However, the optimal WL MVDR beamformer cannot be achieved unless the desired signal’s noncircularity coefficient and SV are known precisely. In practical application, the noncircularity coefficient is difficult to be obtained and the SV is always known inexactly. Hence, some robust WL adaptive beamformers were proposed in [10–12, 14, 15] to solve these problems. In [14], Wang’s method can deal with the uncertainties in the noncircularity coefficient and the array SV, but it is sensitive to the large mismatch of noncircularity coefficient. In [10], Wen et al. proposed the noncircular robust Capon beamformer (NC RCB) to exploit the SO noncircularity of the desired signal and interferences simultaneously. However, this beamformer suffers severe performance degradation at high signal-to-noise ratio (SNR). In [11], the spatial spectrum of noncircularity coefficient is firstly introduced by Xu et al. and is being used to reconstruct the augmented interference-plus-noise covariance matrix (IPNCM). [12] proposed two WL-minimum dispersion based beamforming methods, which can fully use the noncircularity and sub-Gaussian properties of signals. Although the existing beamformers in [11, 12, 15] have good performances, they are difficult to be applied in practical engineering due to their high computational burden.

In this paper, a novel robust WL beamforming algorithm is proposed. Firstly, we construct the augmented IPNCM by using the SVs corresponding to the interference region. Secondly, the reconstructed IPNCM is decomposed to construct the interference subspace, which is being used to preprocess the received data. Thirdly, based on the augmented IPNCM, we apply the oracle approximating shrinkage (OAS) method to estimate the desired signal’s extended SV. Simulation results show that the output signal-to-interference-plus-noise ratio (SINR) of the proposed beamformer is higher than the other WL robust beamformers developed previously in [10–12].

The rest of this paper is organized as follows. The signal model and the new proposed method are described in Section II and Section III, respectively. Our simulation results are given in Section IV. Finally, Section V presents our conclusions.

Notation: Upper (lower) bold face letters are used to denote matrices (column vectors). \( \mathbf{I} \), \( E\{\cdot\} \), \( \|\cdot\| \), \( \text{Tr}(\cdot) \) stand for the unit
orthogonal component of can be rewritten as

\[ s = \text{diag}(X) \]

is the equivalent extended SV of the desired noncircular signal, and \( \tilde{\nu}_n(t) = s_0^H(t)\pi_a(1 - |\gamma|^2)^{1/2}\tilde{a}_2 + \tilde{v}(t) \) is the global noise vector for \( x(t) \). The output of the WL beamformer can be written as \( y(t) = \hat{w}^H \tilde{x}(t) = s_0(t) \hat{w}^H \tilde{a}_2 + \tilde{v}(t) \), where \( \hat{w} \) is the WL weight vector. The optimal WL MVDR beamformer is designed via solving

\[
\min_{\hat{w}} \hat{w}^H R_{\gamma} \hat{w} \quad \text{s.t.} \quad \hat{w}^H \tilde{a}_2 = 1,
\]

where \( R_{\gamma} = \langle \tilde{\nu}_n(t)\tilde{\nu}_n(t)^H \rangle = \begin{bmatrix} Q_r & Q_r' & Q_r'' \end{bmatrix} \) and \( Q_j = \pi_a(1 - |\gamma|^2) \). For the real signal linear, \( Q_j = 0 \). Then, the optimal WL beamformer can be calculated as \( \hat{w}_{opt} = [\tilde{a}^H, \tilde{a}_2]^{-1} \tilde{v} \). In practice, the exact \( \tilde{R}_\gamma \) is unavailable and can be replaced by the extended sample CM \( \tilde{R}_\gamma = (1/L) \sum_{l=1}^L \tilde{x}(t)\tilde{x}(t)^H \), where \( L \) is the number of snapshots.

## III. Our Proposed Method

### A. The augmented IPNCM reconstruction

Firstly, we determine the angular sector \( \Theta \) in which the signal-of-interest (SOI) is located. The sector \( \Theta \) is assumed to be distinguishable from interfering signals, which can be obtained from low resolution direction finding methods [11, 15]. Then, the whole spatial domain \( \Theta_{ad} \) can be divided into two parts, i.e., \( \Theta \) and its complement sector \( \Theta_{ad} = \Theta \cup \bar{\Theta} = \Theta_{ad} \). Let \( \Theta_{ad} \) be the estimated number of signal sources and can be calculated from the \( \tilde{R}_\gamma \) by using the information theoretic criteria based approaches [17], such as minimum description length (MDL).

It is well known that, the beampattern of the MVDR beamformer \( B(\theta_k) = |\hat{w}^H_a(\theta_k)| \) (\( \theta_k \in \Theta_k \), \( k = 1, 2, \ldots, I \)) is expected to form the nulls at the directions of the interferers located in \( \bar{\Theta} \), where \( \theta_k \) is the angles corresponding to these interferers or grating nulls. \( \Gamma \) can be written as

\[ \Gamma = \{ \Gamma : \theta | B(\theta - \Delta \theta) > B(\theta) < B(\theta + \Delta \theta), \theta \in \Theta \}, \]

where \( \Delta \theta \) is the angular sampling interval in \( \bar{\Theta} \). Let \( p(k) \) be the number of the projection of \( a(\theta_k) \) on \( U_\gamma \), i.e., \( p(k) = \| U_\gamma \hat{a}(\theta_k) \| \). We know that the value of \( p(k) \) is large when \( \theta_k \) corresponds to the direction of interference. Then, \( p(k) \) for \( k = 1, 2, \ldots, K \) can be arranged in descending order, as \( p[1] \geq p[2] \geq \cdots \geq p[K] \), and the corresponding \( \theta_k \) are arranged as \( \theta[1], \theta[2], \ldots, \theta[K] \). Whether the input SNR is small or large, the value of \( p \) is greater than or equal to \( P - 1 \) since the interference signals are always strong, thus, the interference angular set \( \Gamma_{in} = \)
\{\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_p\} covers the directions of all interferences, without loss of generality, \(\tilde{\theta}_p\) \((p = 1, 2, \cdots, P)\) is assumed to be corresponding to the \(p\)-th interference’s direction. In addition, sometimes the true DOAs of interferences may not be on the discretized sampling grid, which is called the off-grid DOA estimation problem. However, we can make sure that the true DOA of the \(p\)-th interference is located in the angular area \(\Theta_p = [\tilde{\theta}_p - \delta_\theta, \tilde{\theta}_p + \delta_\theta]\), where \(\delta_\theta\) is a small positive angle with \(\delta_\theta \geq \Delta \theta\). Fortunately, our purpose is to reconstruct the augmented IPNCM of \(x(t)\) rather than estimate the interference’s directions. Hence, we use the angular set \(\Theta_{m} = \Theta_{[1]} \cup \Theta_{[2]} \cup \cdots \cup \Theta_{[p]}\) instead of \(\Gamma_{m}\) to reconstruct the augmented IPNCM, because the spatial power spectrum is mainly distributed in a neighborhood of the actual SV, and its amplitude is very small in other directions [18]. Then, the IPNCM \(Q_r\) and the conjugate IPNCM \(Q_c\) of \(x(t)\) can be reconstructed as
\[
Q_r = \sum_{p=1}^{P} \sum_{m=1}^{M} a(\tilde{\theta}_p, m) a^H(\tilde{\theta}_p, m) + \bar{\sigma}_n^2 I, \tag{8}
\]
\[
\tilde{Q}_c = \sum_{p=1}^{P} \sum_{m=1}^{M} \tilde{\gamma}(\tilde{\theta}_p, m) a(\tilde{\theta}_p, m) a^H(\tilde{\theta}_p, m), \tag{9}
\]
where \(\tilde{\theta}_p, m, m = 1, 2, \cdots, M\) are the sampled angles in \(\Theta_p\) and \(M\) is the number of sampling points, \(\sigma_n^2\) denotes the noise power and can be estimated as the minimum eigenvalue of \(\tilde{\gamma}(\tilde{\theta}_p, m)\). \(\tilde{\gamma}(\tilde{\theta}_p, m) = -\bar{a}^2(\tilde{\theta}_p, m) D a(\tilde{\theta}_p, m) \bar{a}(\tilde{\theta}_p, m) a(\tilde{\theta}_p, m)\) is the spatial spectrum of noncircular interference corresponding to \(\tilde{\theta}_p, m\) [11], where \(D \triangleq (R_x - C_x^T R_x^{-1} C_x)^{-1}\), \(E \triangleq -D C_x^T R_x^{-1}\), \(\hat{n}\) is the minimum eigenvalue of \(R_x\) and \(\hat{C}_x = (1/L) \sum_{t=1}^{L} x(t) x^T(t)\). Finally, the augmented IPNCM can be reconstructed as
\[
R_v = \begin{bmatrix} Q_r & \tilde{Q}_c \\ \tilde{Q}_c^* & Q_c^* \end{bmatrix}. \tag{10}
\]
Compared with the augmented IPNCM reconstruction in [11], the proposed method has a lower complexity since only the SVs corresponding to the interference region are employed.

**B. The desired signal’s extended SV estimation**

In this section, a shrinkage-based approach is developed to estimate the extended SV of the desired signal. Firstly, we preprocess the received data to extract the SOI component. As mentioned above, the interference subspace can be reconstructed as \(U_j = [g_1, g_2, \cdots, g_j]\), where \(Q_r = \sum_{k=1}^{N} \beta_k g_k g_k^H\) and \(\beta_1 \geq \beta_2 \geq \cdots \geq \beta_N\). \(M\) is the minimum integer where satisfies the expression \((\sum_{k=1}^{M} \beta_k)/(\sum_{k=1}^{N} \beta_k) \geq \xi\) and the constant \(\xi\) is used to determine the number of eigenvectors. Then, the subspace orthogonal to the SVs of the interferences can be obtained as \(P_{U_j} = I - U_j U_j^H\). We project the received data onto the \(P_{U_j}\) to get a mixed signal containing only the desired signal and the noise
\[
x(t) = P_{U_j} x(t) = [P_{U_j} a(\theta_0)] s_0(t) + \tilde{n}(t), \tag{11}
\]
where \(\tilde{n}(t) = P_{U_j} n(t)\). We notice that the desired signal’s SV \(a(\theta_0)\) is replaced by \(P_{U_j} a(\theta_0)\). Obviously, in order to recover the SV \(a(\theta_0)\), \(\tilde{x}(t)\) should be multiplied by \(P_{U_j}^T\),
\[
\tilde{x}(t) = P_{U_j}^T \tilde{x}(t) = a(\theta_0) s_0(t) + \tilde{n}(t), \tag{12}
\]
where \(\tilde{n}(t) = P_{U_j}^T \tilde{n}(t)\), and \(P_{U_j}^T\) denotes the pseudo-inverse of \(P_{U_j}\). Subsequently, the extended observation vector of \(\tilde{x}(t)\) can be defined as
\[
\tilde{x}(t) \triangleq [\tilde{x}^T(t), \tilde{x}^H(t)]^T = \tilde{\alpha}_s s_0(t) + \tilde{v}(t), \tag{13}
\]
where \(\tilde{v}(t) = \tilde{n}(t)[\tilde{x}^*(t) + |\tilde{\alpha}_s| s_0(t)]^{1/2} s_0(t)\). Then, a low-complexity shrinkage-based approach is developed to estimate the extended SV of the desired noncircular signal. The cross-correlation between \(\tilde{x}(t)\) and the WL beamformer output can be expressed as
\[
z = E[\tilde{x}(t) y^*(t)], \tag{14}
\]
where \(\tilde{y}(t) = \tilde{w}^H \tilde{x}(t)\) is the WL weighting vector. Assuming that the desired signal \(s_0(t)\), \(\tilde{n}(t)\) and the noise are independent from each other, meanwhile, all signal sources and the noise have zero mean. By substituting (13) into (14), \(z\) can be rewritten as
\[
z = E[\tilde{x}(t) y^*(t)] = \tilde{\rho}_s \tilde{w} \tilde{\alpha}_s + \tilde{v}(t) y^*(t) \tilde{w}. \tag{15}\]
As we know, \(|\tilde{\alpha}_s^H \tilde{w}| \gg |\tilde{v}^H(t) \tilde{w}|\), especially when \(\tau_s \geq 1\), \(|\tau_s \tilde{\alpha}_s^H \tilde{w}| \gg |\tilde{v}^H(t) \tilde{w}|\), thus \(z\) is mainly determined by the first part. Then the more accurate estimate of \(z\) is the better estimate of \(\tilde{a}_s\), we will get.

Next, the OAS method is used to compute \(z\) iteratively. We define \(\tilde{F} = \tilde{w} L\), where \(\tilde{v} = Tr(\tilde{G})/(2N)\) and \(\tilde{G} = \text{diag}(\tilde{x} y^*)\). According to [19], a reasonable tradeoff between bias increase and covariance reduction can be obtained by the shrinkage of \(\tilde{G}\) towards \(\tilde{F}\), and we can apply it in a vector shrinkage form
\[
\tilde{z}(i) = \tilde{\rho} \text{diag}(\tilde{F}(i) + [1 - \tilde{\rho}] \text{diag}(\tilde{G}(i)), \tag{16}
\]
where \(\tilde{\rho}\) denotes the shrinkage coefficient. If we define \(\tilde{Z} = \text{diag}(\tilde{z})\), then the goal is to compute the optimal value of \(\tilde{\rho}\) that minimizes the mean square error of \(E[\|\tilde{Z}(i) - \tilde{F}(i - 1)\|^2]\) in the \(i\)-th snapshot. Finally, (17) and (18) are obtained as below
\[
\tilde{z}(i) = \tilde{\rho}(i) \text{diag}(\tilde{F}(i) + [1 - \tilde{\rho}(i)] \text{diag}(\tilde{G}(i)), \tag{17}
\]
\[
\tilde{\rho}(i + 1) = \frac{(1 - \frac{1}{\tilde{N}}) \text{Tr}[\tilde{Z}(i) \tilde{G}(i)] + \tilde{e}(i)}{(1 + \frac{1}{\tilde{N}}) \text{Tr}[\tilde{Z}(i) \tilde{G}(i)] + (1 - \frac{1}{\tilde{N}}) \tilde{e}(i)), \tag{18}
\]
where \(\tilde{e}(i) = \text{Tr}[\tilde{Z}(i) \tilde{G}(i)]\), and the sample correlation vector \(\tilde{G}(i) = \text{diag}(\tilde{G}(i))\). If the initial value of \(\tilde{\rho}(1)\) is between 0 and 1, the iterative process in (17) and (18) is guaranteed to converge [19]. Based on the equation (5), if we let \(a_1 = a(\theta_0)\) and \(a_2 = \gamma_s a(\theta_0)\), then
\[
\tilde{a}_s = [a_1^T, a_2^T]^T \text{ and } a_2 = \gamma_s a_1. \tag{19}
\]
Meanwhile, the \(\tilde{z}(i)\) can be divided into two parts, i.e., \(\tilde{z}(i) = [\tilde{z}_1^T(i), \tilde{z}_2^T(i)]^T\), where \(\tilde{z}_1(i) \in \mathbb{C}^{2N \times 1}\), \(\tilde{z}_2(i) \in \mathbb{C}^{N \times 1}\), and \(\tilde{z}_2(i) \in \mathbb{C}^{N \times 1}\). According to (15), \(\tilde{a}_s(i) = \mu \tilde{z}(i) = \)
\[ \mu [\hat{z}_2^T(i), \hat{z}_1^T(i)]^T, \] where \( \mu \) is a constant. \( \hat{z}_i(i) \) denotes the estimation of \( z_i \). In the light of the equation (19), if the estimated \( \hat{z}_i(i) \) is accurate enough, then \( \hat{z}_2(i) = \hat{\gamma}_s(i) \hat{z}_1^T(i) \), where \( \hat{\gamma}_s(i) \) is the estimation of \( \gamma_s(i) \). However, this situation will not happen due to the error of estimation, so the estimated \( \hat{z}(i) \) should be corrected. To correct the \( \hat{z}(i) \), we propose to minimize the following cost function

\[
\min_{\hat{\gamma}_s(i)} \| \hat{z}_2(i) - \hat{\gamma}_s(i) \hat{z}_1^T(i) \|^2, \tag{20}
\]

Then, \( \hat{\gamma}_s(i) = \hat{z}_2^T(i) / \| \hat{z}_1^T(i) \|. \) Finally, the extended SV of the desired signal is estimated by

\[
\hat{\nu}(i) = \frac{\hat{z}(i)}{\| \hat{z}(i) \|}, \quad \text{where} \quad \hat{z}(i) = \left[ \begin{array}{c} \hat{z}_1(i) \\ \hat{\gamma}_s(i) \hat{z}_1^T(i) \end{array} \right]. \tag{21}
\]

The steps of the desired signal’s extended SV estimation are summarized in the Algorithm 1. With the estimates for the desired signal’s extended SV, the IPNCM, and the conjugate IPNCM, the proposed WL beamformer can be calculated as

\[
\tilde{\nu}(i) = \hat{\nu}(i) \tilde{R}_v^{-1} \hat{\nu}(i) - 1 \tilde{R}_v^{-1} \hat{\nu}(i), \tag{22}
\]

the inverse of the block matrix \( \tilde{R}_v \) can be calculated as

\[
\tilde{R}_v^{-1} = \begin{bmatrix} \tilde{D}_v & \tilde{E}_v \\ \tilde{E}_v^H & \tilde{D}_v \end{bmatrix}, \quad \text{where} \quad \tilde{D}_v = (\tilde{Q}_v - \hat{\tilde{Q}}_v \hat{\tilde{Q}}_v^{-1} \tilde{Q}_v^{-1}), \end{bmatrix} \]

\[
\tilde{E}_v = -\tilde{D}_v \tilde{Q}_v \tilde{Q}_v^{-1}. \quad \text{In order to show the property of the proposed method, the advantages and drawbacks are summarized in Table I.}
\]

C. Complexity analysis

For the proposed method, the computational complexity of the augmented IPNCM reconstruction is \( O(MPN^2) \), and the computational complexity of the desired signal’s extended SV estimation is \( O((2N)^3) \). Consequently, the overall complexity of the proposed beamformer is \( O(\max\{MPN^2, 8N^3\}) \).

The Xu’s method [11] and the NC RCB [10] have the complexity of \( O(\min\{M\delta N^2, (2N)^{3.5}\}) \), \( (2N)^{3.5} \) respectively. Where \( M\delta \) is the number of sampling points in \( \Theta \). The Huang’s method has a complexity of \( O((L+2N)^{3.5}) \) [12], where \( L \) is the number of snapshots. And the Zhang’s method has a complexity of \( O((2N)^{3.5}) \) [15]. Thus, the proposed method has a lower complexity than the WL beamformers in [11, 12, 15] especially when \( N \) is large. We summarize the computational complexities of the algorithms in Table II.

IV. Simulation Results

In our simulations, a uniform linear array with 10 omni-directional sensors spaced a half-wavelength distance is considered. The additive noise is modeled as complex circularly symmetric Gaussian zero-mean spatially and temporally white process. Two interferences are assumed to be come from the directions 30° and −50°, respectively. The former one is the BPSK with the noncircularity phase −120°. The latter one is the UQPSK with the noncircularity rate 0.8 and the noncircularity phase −150°. Both their interference-to-noise ratios (INRs) are equal to 30 dB. The presumed direction of the desired signal is \( \theta_0 = 5° \). The desired signal’s angular region \( \Theta \) is set to be \( \Theta = [\Theta_0 - 8°, \Theta_0 + 8°] = [−3°, 13°] \), and then \( \Theta = [−90°, −3°) \cup (13°, 90°) \). The proposed beamformer is compared with the Xu’s method [11], the NC RCB with \( \tilde{c} = 3 \) [10], the Huang’s method with \( \varepsilon_\theta = 0.25 \), \( N = 2.5 \) and \( \varepsilon_\nu = 0.1 \) [12]. For the proposed method, the parameters \( M = 15, \tilde{\rho}(1) = 0.8, \xi = 0.98, \tilde{\delta}(1) = 3° \). The sampling angle interval in \( \Theta_{sal} \) is 1°, i.e., \( \Delta \theta = 1° \). In Fig. 1, Fig. 3 and Fig. 4, the number of snapshots is fixed at 50. In Fig. 2 and Fig. 4, the input SNR is fixed at 10 dB. The obtained simulation figures are through 100 independent runs. The output SINR of a filter \( \tilde{\nu} \) is defined by \( \text{SNR}_{\text{P}} = \pi \| \tilde{\nu}^H \tilde{\nu} \| / (\tilde{\nu}^H R_{\nu} \tilde{\nu}) \).

5. In the first example, we study the effect of the signal look direction error on the performance of the tested beamformers. The desired signal is the UQPSK with the noncircularity rate \( \gamma_s \) = 0.8 and the noncircularity phase \( \phi_s = 60° \). In Fig. 1 and Fig. 2, the random DOA mismatch of the desired signal and the interferences are uniformly distributed in \([-4°, 4°] \), where the random DOAs change from run to run but remain

\[
\text{TABLE I ADVANTAGES AND DRAWBACKS OF THE PROPOSED METHOD}
\]

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. High output SINRs for the reception of noncircular signals, and maintaining a good performance over a wide range of SINRs.</td>
<td>1. The angular sector of the desired signal should be a priori, and the interferences must be outside the main beam.</td>
</tr>
<tr>
<td>2. Not need a priori knowledge of the noncircularity coefficient of the desired signal.</td>
<td>2. Cannot process more interferences than the number of sensors.</td>
</tr>
<tr>
<td>3. Low computational complexity.</td>
<td>3. Performance degradation for calibration errors.</td>
</tr>
</tbody>
</table>

\[
\text{TABLE II COMPUTATIONAL COMPLEXITIES OF ALGORITHMS}
\]

<table>
<thead>
<tr>
<th>Proposed</th>
<th>( O(\max{MPN^2, (2N)^3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhang's</td>
<td>( O((2N)^3) )</td>
</tr>
<tr>
<td>Huang's</td>
<td>( O((L+2N)^{3.5}) )</td>
</tr>
<tr>
<td>Xu's</td>
<td>( O(\min{M\delta N^2, (2N)^{3.5}}) )</td>
</tr>
<tr>
<td>NC RCB</td>
<td>( O((2N)^3) )</td>
</tr>
</tbody>
</table>
Fig. 1. Output SINR of beamformers versus input SNR in the case of look direction mismatch.

Fig. 2. Output SINR of beamformers versus the number of snapshots in the case of look direction mismatch; SNR = 10 dB.

fixed from snapshot to snapshot. The output SINR versus the input SNR and the number of snapshots are shown in Fig. 1 and Fig. 2, respectively. According to Fig. 1, when SNR ≤ −5 dB, the performances of all methods are pretty much the same, and when SNR ≥ 0 dB, the proposed method obtains a higher output SINR than the other methods. The NC RCB and the Huang’s method suffer from severe performance degradation when the look-direction mismatch of the desired signal is ≥0, and when SNR ≤ −5 dB, its output SINR is very close to the optimal WL MVDR.

In Fig. 4, we study the performance of all the methods when the look-direction mismatch of the desired signal is increasingly further apart. The simulation conditions in Fig. 4 are the same as Fig. 1 except that the desired signal’s actual direction $\theta_0$ varies from $4.5^\circ$ to $0^\circ$ with a step size of $−0.5^\circ$ (the look-direction mismatch $|\theta_0 − \theta|$ varies from $0.5^\circ$ to $5^\circ$). The DOAs of the two interferences are fixed at $0^\circ$ and $5^\circ$, respectively. It can be seen from Fig. 4 that the output SINR
of the proposed beamformer is higher than the others over a wide range of look-direction mismatch. Nevertheless, when the look-direction mismatch is greater than 3°, the performance of the proposed algorithm is shown to be drifted apart from the optimal output SINR due to the error of the estimation.

V. CONCLUSIONS

In this paper, we have developed a WL beamforming algorithm to robust against the desired signal’s SV mismatch for noncircular signals. The augmented IPNCM is reconstructed first based on the spatial spectrum of noncircularity coefficient, and then a modified OAS method is developed to estimate the desired signal’s extended SV. Simulation results have been presented to demonstrate the effectiveness of the proposed method.

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